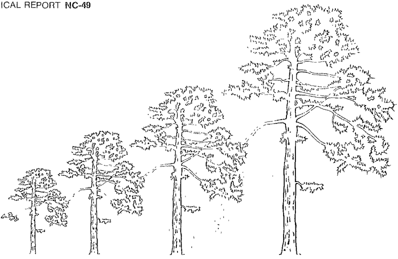
USDA FOREST SERVICE
GENERAL TECHNICAL REPORT NC-49



A GENERALIZED FOREST GROWTH PROJECTION SYSTEM

Applied To
The Lake States Region

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North Central Forest Experiment Station
Forest Service, U.S Department of Agriculture

PREFACE

The papers presented here describe a new computerized system developed to project forest growth and mortality with or without tree cutting for any mix of species or stand structure encountered in the field.

Growth and mortality coefficients have been developed for the forests of the Lake States Region, and the system has been used to update forest inventories, identify silvicultural treatment opportunities, and project long-run timber output and growth response. The system is designed to be used as the core of a larger Forest Resources Evaluation Program (FREP) being developed by the North Central Station, but it has potential for much wider use.

Work to enhance and refine the system's capabilities is continuing, and further progress has been made since these papers were prepared for publication. Interim modules to account for regeneration and for tree cutting to meet timber product removal objectives have been added. Although the capabilities of the projection system are changing rapidly we thought it appropriate to publish these papers to give a snapshot picture of its development as of September 1977.

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1979

FOREST RESOURCES EVALUATION SYSTEMS— A NEEDED TOOL FOR MANAGING RENEWABLE RESOURCES

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and **Burton L. Essex**, *Principal Resource Analyst*

The Forest and Rangeland Renewable Resources Planning Act of 1974 requires the Secretary of Agriculture to prepare a Renewable Resource Assessment for the Nation every 10 years, and a long-range Renewable Resource Program for the Forest Service every 5 years. The first assessment and program were published in 1976 (USDA Forest Service 1976a, 1976b). The second assessment is scheduled to be made during 1979, with a second program document based on this assessment due in 1980. These resource assessments provide the basis for national programs, but also can serve as guides for framing regional resource policies and programs.

State land management and planning agencies require resource assessments at the state and local level for long-range resource planning and development. River basin and other special-area planners need resource inventory information for political subdivisions updated to a common base year.

Large individual private and public forest properties and management units likewise need resource evaluation systems designed to project prospective supply and demand from their properties, and to identify and evaluate potential management opportunities.

NORTH CENTRAL'S RESPONSIBILITY

The North Central Forest Experiment Station of the Forest Service, U.S. Department of Agriculture, has the responsibility for conducting statewide forest resource inventories and regional resource assessments within the North Central Region of the United States. To meet these and other needs expressed above, the Station must have the capability to acquire, process, and evaluate information about the current and future extent, condition, and use of the renewable resources within the Region. An evaluation system must be designed specifically to fit the existing data base and data analysis capabilities already existing for this Region. This system must be usable for regional, State, and local assessments as the need arises, and be compatible with any national assessment system developed.

WHAT SHOULD A RESOURCES EVALUATION SYSTEM DO?

Obviously, we need an evaluation system that can, using basic forest resource inventory and other data available for a resource area, project future forest resource supply and demand for the area, and evaluate potential alternative resource management programs designed to change supply or demand. Such a system or series of systems should be able to evaluate timber, wildlife, recreation, water, forage, and special uses of the forest resource and their interactions from an ecologic-environmental, economic, and socio-cultural point of view. There is little doubt that such a resource evaluation system must be computer-based if it is to handle the large amount of information and analytical procedures necessary for regional assessment.

The dimensions of this job are enormous. They include economic, ecologic-environmental, and socio-cultural evaluations of the following resource use systems: timber, wildlife, recreation, water, forage, and special uses (fig. 1). For each resource

Type of Evaluation	Resource Use System					
	Timber	Wildlife	Recreation	Water	Forage	Special uses
Ecologic						
Environmental						
Economic						
Social						
Cultural						
Legal						

Figure 1.—*Potential dimensions of a forest resources evaluation program.*

use system we must obtain and organize the physical and biological information needed for evaluation, and develop methods for doing this within reasonable cost and time constraints. We also must develop and apply methods for evaluating economic and social costs and benefits of defined alternatives. Substantial work is needed to determine the nature and extent of interrelations among the various resource use systems so that these can be considered in evaluating opportunities and making regional assessments. These analyses should be made for the different resource components, resource owners, owner organization levels, and resource users (both direct and indirect), and for different spatial and temporal extents. They also should recognize the degree of uncertainty regarding our knowledge.

Because the existing forest resource is usually in a condition far from ideal, such an evaluation system must be capable of dealing with the resource situation as it exists on the ground. It must be able to handle the entire range of sites and stand conditions, all levels of tree stocking, cull and sound trees, and almost every conceivable mixture of tree species and sizes. It must be able to produce both short- and long-run projections of resource conditions for a wide range of potential management treatments within an acceptable level of accuracy. Further, it should be able to handle the other components of forest ecosystems besides trees in order to cover all forest uses.

EXISTING SYSTEMS ARE INADEQUATE

A forest growth projection system capable of handling the complexities of the real world in enough detail to evaluate alternative stand treatment programs has not been available in the past. Individual growth and yield models have been developed, but mainly for pure stands of single species or for a few types under limited conditions. These, however, have proven to be inadequate in dealing with the mixed ages, sizes, and species, the non-normal stocking, or the high proportion of cull trees that often typify existing stands. Because each model was developed independently, using different kinds of inputs and producing different kinds of output, attempts to incorporate them into a comprehensive analytical system have not been successful. Also, since no growth and yield models exist for many forest types and stand conditions, any system developed from existing models would be, at best, incomplete.

It is doubtful that a comprehensive system capable of conducting integrated multi-resource, multi-use, multi-objective evaluations simultaneously will be successfully developed in the near future. Rather than wait until a complete system is developed, the North Central Station has decided to develop parts of the system now. Since much work already has been done on the timber resource, developing a computerized system to evaluate the timber resource in economic terms was chosen as a logical first task. Later, other resource components will be linked to timber.

WHAT'S THE FIRST STEP?

As a first step in meeting the need for a regional forest resource assessment system, we are developing a computerized timber resource projection and evaluation system that will update and project timber inventories and product yields, determine the extent of potential stand treatment opportunities, and evaluate from an economic standpoint potential programs to increase future timber supplies. Figure 2 shows the major components of this system that require development. Each of these components in turn includes subcomponents that will require considerable research.

THE GROWTH PROCESSOR—NUCLEUS OF THE RESOURCES EVALUATION SYSTEM

Basic to this timber resource evaluation system is a subsystem to project future stand development in response to proposed management activities. To be useful for regional analyses, such a forest growth projection system must handle all forest conditions encountered in forest inventories. In short, it must be a generalized forest growth projection system.

This growth projection system must project future forest growth on forest inventory plots so they can be updated to any given base year. To do this adequately it must account for growth on all trees inventoried, mortality, recruitment or ingrowth of new trees, and removals of trees through cutting. It must cover the entire range of site and stand conditions encountered.

This growth projection system also must project the response of individual stands or classes of stands to prescribed treatments, such as precommercial thinning, partial cutting, and cull-tree removal, so treatment opportunities can be evaluated. When fully developed, it will also be used to estimate potential yields from fully-stocked stands to establish the potential productivity of specified sites. Such a system is basic to many types of timber resource evaluations.

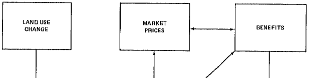

Figure 2.—*Major components of a timber resource evaluation system for economic analysis.*

NORTH CENTRAL'S APPROACH

In recognizing the need for a new approach to this problem, researchers at the North Central Forest Experiment Station have developed a computer-based, generalized forest growth projection system that can be used as the basis for a timber resource evaluation system. The details of this system are summarized in the papers that follow. Although the system is still under development, it already can project short-run growth responses in enough detail to be useful in evaluating treatment alternatives in many stands. With further development it will provide the basis for a timber resource evaluation system for regional forest resource assessments.

LITERATURE CITED

USDA Forest Service 1976a. RPA: The nation's renewable resources—an assessment, 1975. 345 p. Washington, D.C.

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DESIGN

Rolfe A. Leary, *Principal Mensurationist*

A major new direction is surfacing in the development of forest growth models. The trend is away from single-species models toward models sufficiently general to handle any pure or mixed stand of any structure and density on any site. This paper explains the design features of a generalized forest growth projection system that can be used for short- to medium-range projections of existing stands. The details are found in other chapters of this report.

Development of mathematical equation models of forest growth has been an active area of mensuration research for the past 40 years. One of the early studies reported was that of MacKinney *et al.* (1937). In the early work, goodness of fit of model to data was the primary discriminating variable in model selection. Grosenbaugh (1958) developed this to a highly detailed state. Little consideration was given to generality or theorification potential. Most models applied only to pure stands of a single species with relatively simple stand structures. The last 5 to 10 years have seen many efforts to develop growth models general enough to handle a variety of conditions (Fries 1974). The models for some species are quite detailed, giving a high resolution level, but other models, especially for economically unimportant species and mixed stands, are still at low resolution levels. Several approaches (e.g., Stage 1973, Ek and Monserud 1974, Hegyi 1975, and probably others[1]) have been developed and are being improved and extended to cover additional forest conditions.

DESIGN OBJECTIVES

I had two fundamental objectives in designing the model part of this growth projection system. The first was to capture, to the degree possible, the "essence" of the process of stand growth and development. Thus, I sought a structural similarity between the model and the forest stands. The second objective was to accomplish the first objective as efficiently as possible.

From the standpoint of modeling, a forest stand can be viewed as a set of interactive, differential energy transformers and accumulators (Leary 1970b). Thus, to model forest stands, one must deal effectively with these five key concepts: (1) set, (2) interactive, (3) differential, (4) transformer, and (5) accumulator. Nearly all models address these concepts, although they may be given different names or not be identified explicitly.

From "set" we know we are dealing with an aggregate of individuals; from "interactive" we know the individuals in the aggregate are interacting; from "differential" we know that not every individual in the set is the same; from "transformer" and "accumulator" we know that the individuals are both factory and warehouse. If we can develop a mathematics that will address each of the concepts and synthesize them into a whole, we may have some structural similarity between model and nature.

My preference was to look first to "transformer" and "accumulator" when shopping for a mathematics to use. This nature-based constraint on our shopping list directs us to differential equations. For the sake of simplicity we go to first-order ordinary differential or difference equations. Here we see the symbolic expression of this ideal:

$$\frac{dY}{dt} \text{ or } \frac{\Delta Y}{\Delta t} = f(Y, \ldots)$$

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[1] *Arney, J. S. 1977. Personal communication. Centralia, Washington.*

Verbally these symbols are saying: "that which results from growth is itself typically capable of growing". It is important to understand that Y is the dependent variable in this equation and t the independent. Persons accustomed to using regression methods sometimes have difficulty, especially with the differential equation form, because the dependent variable, Y, is on both sides of the equality.

Once having made the critically important choice of a mathematics to use, we can proceed to examine the possibilities for dealing with "set", "interactive", and "differential". The concepts of set and differential are complementary in the following way: from set we easily consider subsets, but not just any subsets. They should be structured in a way so the individuals within a subset are similar and those between subsets are different. Subsetting or stratifying an aggregate is a natural way to formalize differentia. Thus, the union of concepts "set-subset" and "differential" suggest that we may need as many Y's as we have subsets. We turn, of course, from scalars to vectors to accommodate this need. The result, in terms of a symbolic expression of our ideal, is now only slightly altered:

$$\frac{dY1}{dt} \text{ or } \frac{\Delta Y1}{\Delta t} = f(Y1, Y2, \ldots)$$

$$\frac{dY2}{dt} \text{ or } \frac{\Delta Y2}{\Delta t} = f(Y2, Y1, \ldots)$$

These equations must be solved together because they are coupled or simultaneous equations. We see that the mathematics of differential/difference equations is so well-suited to our problem that in the handling of "set-subset" and "differential" we have also dealt with the concept of "interactive".

The practicing forester often considers "stand" through more operational concepts: (1) structure, (2) density, (3) species composition, (4) site, and (5) stand history. How does the mathematics we have selected allow us to account for these important aspects of "stand"? The later sections of the paper go into the details of this question, so here I will only indicate their general occurrence in the above equations.

$$\begin{array}{l} \dfrac{\Delta Y1}{\Delta t} = f(\text{density, site, stand history}, \ldots) \\[2ex] \dfrac{\Delta Yn}{\Delta t} = f(\text{density, site, stand history}, \ldots). \end{array} \quad \updownarrow\text{STRUCTURE} \quad \text{SPECIES} \quad \updownarrow\text{COMPOSITION}$$

Thus, we see that density, site, and stand history are dealt with by making them arguments in the right-hand side of a difference equation. Stand structure and species composition are accommodated by changing the number of simultaneous equations.

An important operational advantage made possible by our choice of the mathematics of difference equations is the ease with which resolution level may be handled. The next few sections of the paper are organized around the concept of resolution level. They reflect the current design state, and are in part chronological because a conscious method used was one suggested by Polya (1945): take an insoluble (but apparently simple) problem and embed it into a larger problem so as to create a class of problems. The reasoning is that the solution to the larger class of problems may be more obvious than the solution to the one simple problem.

For example, the potential choices of a model for a single-species stand with unimodal diameter distribution are numerous. Generalization to handle mixed stands with varying stand structures and densities is possible with some and not with others. This eliminates a large fraction of the candidate models. Other forms are eliminated by applying the reverse process, and assessing the ability to reduce or collapse the complicated model to a simple form for simple stands and less precise estimates. The result of this process is a tremendous reduction in the number of feasible models that can make up the core of a generalized projection system capable of variable resolution levels.

HOW THE PROJECTION SYSTEM WORKS

Like any system, the workings of the projection system may be broken up into three parts: input, output, and transformation or internal workings.

Part	*Form*
input	a list of interacting trees showing species, d.b.h., and crown ratio[2] for each tree.
output	a list of trees showing species, new d.b.h. and new status if tree is a mortality tree.
transformation	a stand component change equation, an allocation rule, and a mortality function.

Input and Output

The input list must identify each tree's species and d.b.h. and hopefully its crown ratio. Although many temporary inventory procedures do not record trees in this detail, it is possible to fabricate a tree list from a stand table that would, if summarized, produce the stand table. Crown ratio is collected by the national Forest Survey, but by few others. This important, although usually missing, input variable has been successfully approximated through a simple relation between mean crown ratio and 10-year mean basal area density (Holdaway *et al.* 1979). We use crown ratio as an indicator of stand history. It helps to tell us whether a stand with 80 square feet of basal area grew up from 40 square feet or was cut back from 160 square feet.

Output from the projection system is a new set of d.b.h. values for the trees which survived the projection interval and a status change from live to dead for all trees projected to die.

Although the input tree list is aggregated to form initial conditions for the system, the tree list is not discarded. This would be throwing away information. Instead, it is carried along and updated every year or periodically. This greatly facilitates a flexible output of information. The job of summarizing the output tree list is thereby separated, as it should be, from the job of summarizing the input tree list and growing the stand. Output tree list diameter class breakdowns should not be forced on the projection system. They are normally based on technology and economics, and are of no value in projection.

Transforming Input to Output

The mathematical model for transforming the input tree list to an updated output tree list has four parts: potential function, modifier function, allocation rule, and mortality function.

The first two parts fit together as shown below:

$$\left[\begin{array}{l}\text{One year change}\\ \text{in sum of tree}\\ \text{diameters for a}\\ \text{given number}\\ \text{of trees}\end{array}\right] = \left[\begin{array}{l}\text{potential yearly}\\ \text{d b h growth}\\ \text{for a tree}\\ \text{with mean}\\ \text{crown ratio}\\ \text{and dbh on a}\\ \text{given site}\end{array}\right] \cdot \left[\begin{array}{l}\text{number}\\ \text{of trees}\end{array}\right] \cdot \left[\begin{array}{l}\text{fraction of the}\\ \text{potential growth}\\ \text{actually}\\ \text{occurring}\end{array}\right]$$

In symbolic equation form the relationship is:

$$\frac{\Delta Y}{\Delta t} = [PDG]\cdot[NT]\cdot[MODIFIER] \qquad (1)$$

The potential function is designed to estimate how rapidly the mean tree would be growing in d.b.h. if it were not interacting with any other trees (Hahn and Leary 1979). The potential growth is multiplied times the number of trees, giving a potential change in sum of diameters for NT trees. The modifier function reduces the potential change to what has been observed from permanent growth plots (Leary and Holdaway 1979). In somewhat more detail, and with a slightly changed notation, equation (1) is:

$$\frac{\Delta Y}{\Delta t} = [f_1(\overline{D},\overline{CR},SI)][NT][f_2(Y,NT)], \qquad (2)$$

where:

f_1 is the potential function,
f_2 is the modifier function,
$\overline{D}$ is mean d.b.h.
$\overline{CR}$ is mean crown ratio,
SI is site index,
Y is sum of tree diameters, and
NT is number of trees.

The allocation rule disaggregates the projected growth put on by NT trees, $\Delta Y/\Delta t$, to the individual trees on the input tree list. The simple rule:

$$\left[\begin{array}{l}\text{percent of } \Delta Y/\Delta t\\ \text{that should be given}\\ \text{to tree } i\end{array}\right] = f\left[\begin{array}{l}\text{percent of growing}\\ \text{stock represented}\\ \text{by tree } i\end{array}\right] \qquad (3)$$

has proven to be a useful point of beginning for this task (Leary, *et al.* 1979).

The mortality rule takes the allocated d.b.h. growth for each tree and, using an appropriate function, computes a probability of death for a tree of that species growing at the allocated rate (Buchman 1979). Equation (4) shows this basic relation:

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[2] *Crown ratio is the ratio of full live crown length to total tree height.*

$$\left[\begin{array}{l}\text{Probability of}\\ \text{death for } i^{th}\\ \text{tree of species } j\end{array}\right] = f\left[\left(\begin{array}{l}\text{allocated d.b h}\\ \text{growth for previous}\\ \text{period}\end{array}\right), \left(\text{species}\right)\right] \cdot (4)$$

The computed probability is compared with a pseudo-random number drawn from a uniform distribution on the interval [0,1]. If the random number is less than the computed probability, the tree is called a mortality tree, and its status changed to reflect this. It does not enter further in the projection computations.

Once the mortality has been specified and the tree list otherwise updated, it is again summarized for input, and equations (2), (3), and (4) are executed to project another year's growth. The entire process is repeated over and over again as many times as the user desires.

Throughout this discussion it has been implicit that all trees in the stand would be grouped together and projected with a single equation. Thus, at this level of resolution the model is at the stand model end of the stand model-tree model continuum of growth models.

In moving away from the stand model we go to stand component models, wherein groups of trees in the stand are projected with different equations. The key to this flexibility lies in the use of simultaneous equation models and in the particular form of the modifier function. The potential function, allocation rule, and mortality functions are little affected as resolution level is changed.

LOW RESOLUTION MODEL STRUCTURE

When the projection system is operating at a low resolution level, the input tree list is summarized into two or more groups called components. For this discussion we assume that tree species is the breakdown criterion. If one has, for example, a two-species mixture, the input tree list is summarized into two components. Each component is projected by one of a system of two simultaneous equations as follows:

$$\Delta Y1/\Delta t = \left[\begin{array}{l}\text{Potential d b h.}\\ \text{growth of mean}\\ \text{tree of first species}\\ \text{as a function of}\\ \overline{D}_1, \overline{CR}_1 \text{ and } SI_1\end{array}\right] \cdot \left[\begin{array}{l}\text{number of}\\ \text{trees of}\\ \text{first species}\end{array}\right] \cdot \left[\begin{array}{l}\text{fraction of potential}\\ \text{that first species}\\ \text{attains due to pres-}\\ \text{ence of trees of}\\ \text{species one and}\\ \text{species two}\end{array}\right] \quad (5)$$

$$\Delta Y2/\Delta t = \left[\begin{array}{l}\text{Potential d b h}\\ \text{growth of mean}\\ \text{tree of second}\\ \text{species as a}\\ \text{function of}\\ \overline{D}_2, \overline{CR}_2 \text{ and } SI_2\end{array}\right] \cdot \left[\begin{array}{l}\text{number of}\\ \text{trees of}\\ \text{second}\\ \text{species}\end{array}\right] \cdot \left[\begin{array}{l}\text{fraction of potential}\\ \text{that second species}\\ \text{attains due to pres-}\\ \text{ence of trees of}\\ \text{species two and}\\ \text{species one}\end{array}\right]$$

$\Delta Y1/\Delta t$ is disaggregated to trees of the first species using a species-specific allocation rule, and $\Delta Y2/\Delta t$ to trees of the second species. Mortality probabilities are computed with species-specific coefficients in the mortality equation. An updated tree list is then ready to be summarized again and projected for another period.

Let us go back and take a closer look at the modifier function, since it, of all parts of the model, is changed the most. The functional form of the modifier used to date has been:

$$[1 - e^{\beta / f(Y, NT)}] \cdot \qquad (6)$$

The form in (6) is used when the model is for a single species, hence has a single equation—the lowest resolution level possible. Here β represents how the species interacts with itself in a pure stand. When going to a mixed stand, β is not changed, but the denominator of the exponent is, as follows:

$$\frac{\Delta Y1}{\Delta t} = [PDG_1]\,[NT_1] \left[1 - e^{\beta_1 / (f_1(Y1, NT_1) + \omega_1 f_2(Y2, NT_2, Y1, NT_1))}\right]$$

$$(7)$$

$$\frac{\Delta Y2}{\Delta t} = [PDG_2]\,[NT_2] \left[1 - e^{\beta_2 / (f_1(Y2, NT_2) + \omega_2 f_2(Y1, NT_1, Y2, NT_2))}\right],$$

where

β_1, β_2 represent how species one and two, respectively, affect themselves when growing in pure stands,

f_1, f_2 are functions of sum of diameters (Yi) and number of trees (NT_i) of the species being projected, and

ω_i are the coefficients that tell how species two affects species one (first equation) and species one affects species two (second equation).

If $\omega_1 = 1$, this indicates that species two affects species one the same as species one affects itself. A similar interpretation may be given to $\omega_2=1$. The specific forms of the functions f_1 and f_2 in equation (7) are discussed in detail elsewhere (Leary and Holdaway 1979).

INTERMEDIATE STRUCTURE

Intermediate level models differentiate not only by species but also by size within species. Thus, the input tree list is partitioned by size class within species. How do we incorporate size of tree into the model?

There are three separate but related aspects to this question. One has to do with the mechanism by which the model handles size in addition to species. The second aspect deals with the class movement problem — the movement of trees from one size class to another. The third deals with choosing class bounds for aggregating the input tree list.

The first aspect is easily handled because the model form is such that equations may be added to the system to project size classes within species. When this is done the denominator of the exponent in the modifier function is more complex, but the potential function, allocation function, and mortality function remain essentially unchanged. The modifier function for the first equation would take on a form such as:

$$1\text{-}e^{\beta_1/(f_1\,(Y1,NT_1)\,+\,\omega_{11}\,f_2\,(Y2,NT_2,Y1,NT_1)\,+\,\omega_{12}\,f_3\,(Y3,NT_3\,Y1,NT_1)\,+\,\dots)} \tag{8}$$

The denominator of the exponent of e thus has the form of a linear combination of interaction coefficients and functional relations between the component being projected and all other components. This pattern of relations (8) is repeated for each size class within each species, that is, for each equation in the system.

The second aspect, how to move trees from one class to another, is dealt with by keeping all trees in the class to which they were initially assigned. Since the trees in a class are growing, the attributes of the class change over time, rather than the trees changing classes. Two substantial benefits result from this approach: (1) Because trees do not change classes, summaries of permanent growth plot data do not have discontinuous trends caused by trees growing into and/or out of a class. The summaries, such as basal area in a class at several points in time, show fairly continuous changes. This allows modern calibration methods to be used to estimate the β and ω values (Leary 1970a, 1970b, 1972, 1975; Leary and Skog 1972). (2) Computational efficiencies are gained in the projection process itself since it is not necessary to check the correctness of class assignment as the trees' diameters are increased. An added benefit is that the user has the choice of not applying the allocation rule-mortality submodels every year, rather only when output is desired.

The third aspect is treated by partitioning the list of trees of each species into relative size groupings rather than into size groupings set on a cardinal scale of numbers. If, for example, we use two size equations for each of two species, the d.b.h. class limit might be such that a predetermined percent of the trees are larger than the limit and the rest smaller. By using this relativistic approach when calibrating the modifier function, each equation of the simultaneous system is assured of having trees present in the class it represents. In using the model for projection, we avoid the computational inefficiencies incurred when many equations (size classes) have no trees in them. Of course, mortality or partial cutting could cause some of the relativistic classes to have few, if any, trees, but this can always be rectified by reallocating the trees to the classes.

The increased precision that occures from adding more size equations is not known at this time. It is anticipated that three equations per species will meet many needs.

The allocation rule works as before except that the percentages are based on each component-that is,

$$\begin{bmatrix}\text{percent of } \dfrac{\Delta Y12}{\Delta t}\\ \text{that should be}\\ \text{given to tree } i\end{bmatrix} = f\begin{bmatrix}\text{percent of growing}\\ \text{stock in component 1,2}\\ \text{represented by tree } i\end{bmatrix} \tag{9}$$

Again, the workings of the mortality submodel are unaffected by resolution level. It functions the same as for the simplest model.

HIGHEST RESOLUTION MODEL STRUCTURE

Let's skip now to the highest resolution level that this model can produce. It is attained when the number of simultaneous equations is increased until each tree is in a class by itself, hence projected by a separate equation. In this way it is clear that there is a continuum of growth models from stand (all trees in one class) to tree (one tree in each class).

In going to the highest resolution level the basic model structure is the same:

$$\frac{\Delta Y_k}{\Delta t}=[PDG_k]\,[1]$$

$$\left[1\text{-}e^{\beta/(f_1\ (Y_k)+\sum\limits_{j=1}\omega_j f_2\ (Y_k,\ Y_j,\ x_k-x_j,\ y_k-y_j))}\right].(10)$$

Here the denominator of the exponent of e has the same formulation as many individual tree growth models. Y_k is now the diameter of the subject tree with spatial coordinates x_k,y_k. The neighborhood trees are of diameter Y with spatial coordinates x_j,y_j. The function f_2 characterizes the competition factor common to these formulations.

The potential function now predicts the maximum d.b.h. growth of a single tree with diameter Y_k, crown ratio CR_k, on site, SI. The allocation rule is no longer needed because in equation (10) tree *i* is 100 percent of change as well as growing stock. It can be retained, however, by making the functional relation in (9) the identity relation. The mortality function and relation operate normally, giving an estimate of the probability of death based on $\Delta Y_k/\Delta t$ rather than the disaggregated growth given by the allocation rule.

EFFICIENCIES OF THE DESIGN

Efficiency of the design may be viewed from a number of perspectives. Consider the following:

(a) Efficiencies in modeling are permitted by breaking the overall problem into smaller, semi-independent parts.

When the overall model is decomposed into the potential, mortality, and modifier functions, and an allocation rule, efficiencies result because all but one can be separately completed prior to a synthesis of the parts. Team efforts need this parallelism. The relation of the jobs to each other and to the data base is shown in figure 1 for the present, nearly parallel effort, and for a hypothetical serial effort. Figure 1a shows that only the modifier function development is dependent on completion of another part before it can be totally completed. This allows a rapid updating of coefficients in all models once a new or updated calibration data base is available. It also minimizes the amount of work needed to recover from errors. Figure lb shows a serial or cumulative type of model development. This approach has two major disadvantages: submodel 4 cannot be completed until submodel 3, and so on, and an error in submodel 1 requires changes in submodels 2, 3, and 4 as well as in itself. Both of these disadvantages relate to the speed with which work can be accomplished. Models of a serial or cumulative nature are best constructed by one person.

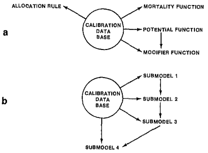

Figure 1.—*(a) Relation between data base and submodels for current design showing a nearly parallel structure. (b) Relation between data base and submodels for a hypothetical design showing a serial structure.*

(b) Efficiency is related to use of information in the calibration data base.

Recall that a design feature of the model calls for each tree to be assigned to a particular class and to remain in that class, and for the size distribution of trees to be partitioned so that about the same number of trees is in each class. When these two features are combined it is possible to use a multipoint boundary value problem approach to the estimation of the modifier function. The source of efficiencies here is that a boundary value approach does not require the taking of differences, often over uneven time intervals (see table 2, Christensen *et al.* 1979). Numerical analysts caution against taking differences of a noisy signal because doing so accentuates the noise relative to the true signal. The regression approach, wherein differences are taken, admittedly used in the potential, mortality, and allocation rule, ignores this caution. Keeping a small noise-to-signal ratio is very important in regions of the country where forest growth rates (i.e., signal strengths) are relatively low.

When the input tree list is partitioned by absolute size class limits, for example, 4.0-5.9, 6.0-7.9, etc., in a stand table projection, numerous stands to be projected may have no trees in the upper or lower classes. When the input tree list is partitioned on a percentile basis, each size class is certain of having a number of trees present. This ensures that the calibration data base information available to develop the projection equation coefficients is used on every plot. Figure 2 compares the traditional stand table and relativistic approach to tree list partitioning.

(c) Efficiency is related to simplicity of the mathematical equations.

When using differential/difference equations such as dY/dt or ΔY/Δt=f(Y) to explain a pattern of Y over time, also given by Y=g(t),the rule is that f is algebraically simpler than g. In fact, quite simple functions, f, can produce an extremely varied pattern of Y values over time. The use of systems of difference equations to handle the process complexity introduced by mixed tree sizes and species helps to keep the right-hand side of each equation as simple as possible.

(d) Efficiency is gained by building into the model the response to treatment.

The particular form of the right-hand side of the difference equations helps answer the question of stand response to a treatment calling for tree removals. This is made possible by having an upper limit of component change, given by the product of potential growth and number of trees, and a modifier of the upper limit, expressed as a function of stand density. The modifier has a form such that as density increases by growth, the value of the modifier decreases, but if density is abruptly decreased by cutting or mortality, the modifier is increased (fig. 3).

(e) Efficiency of computations must also be considered.

Here the concern is the number of arithmetic operations that must be performed per unit of output information; for example, an updated tree list. The multiple resolution level design feature is responsible for most of the gains here. If one wishes, the number of equations may be reduced to one (for a pure stand), and the tree list diameters updated every 10 years rather than every year. Doing this would have an effect on the precision of estimates of future forest conditions, but it would materially reduce the computational load to produce an updated tree list. The trade-offs between computational load and precision of estimates have not been thoroughly examined at this time.

(f) Efficiency in computer hardware requirements should be a part of the design.

The design of this projection system is such that one is not restricted to either a large storage capacity or extremely fast digital computer. Storage requirements can be reduced by such measures as:

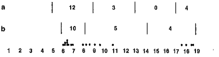

Figure 2.—*(a) The traditional (metric) approach to partitioning a sample distribution of trees. Notice that the 12.9-16.9-inch class has no trees. (b) The relativistic approach to tree list partitioning used in this Design. For illustration the groupings are nearly ½, ¼, ¼*

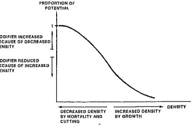

Figure 3.—*Relation between changes in the proportion of the potential change, increased density due to growth, and decreased density due to mortality and cutting.*

(1) projecting shorter tree lists (i.e., smaller plots), and (2) using lower resolution levels. Execution times can be reduced by employing both of these measures and by updating tree diameters and estimating mortality every 10 or so years (rather than every year). By a judicious combination of measures to reduce both storage and execution time requirements, it is anticipated that the projection system can be made to function on both full-size and mini-computers.

DIMENSIONALITY

The fundamental premise in calling the higher dimensional models higher resolution models is based on the reduction in heterogeneity within class as class "width" is reduced. Theoretically, as heterogeneity is reduced, ability to predict growth is increased. In a practical sense, however, the increased predictability may not be a linear function of number of classes (equations). In other words, going from a single equation for a two-species mixed stand to two equations may increase precision 20 percent, but adding two more equations may not increase precision another 20 percent. We are confronted with the costs of calibrating and using whatever resolution level is developed. The determination of an optimum resolution level for growth modeling seems analogous to finding optimum sampling intensities wherein sampling intensities have precisions and costs associated with each.

The use for which a growth projection system is intended, it has been argued, determines the resolution level at which one should operate. Another consideration is the following relation between dimensionality of the model (equivalent to my resolution level) and the time horizon over which projections are to be made:

$$d = f\left(\frac{1}{h}\right) k_a \tag{11}$$

where h is the time horizon over which projections are to be made,
d is the dimension of the model state vector,
k_a is a constant for a given accuracy or precision level, and
f is a function (Kahne 1976).

For simplicity's sake, if f is assumed the identity function the relation is:

$$d = \frac{k_a}{h} \tag{12}$$

Thus, the model dimension should be reduced for long-range projections and increased for short-range projections, to give the same level of accuracy.

The relation in equation (11) is apparently well-accepted by scientists involved in large-scale modeling. In general, however, it is not accepted by scientists working in forest projection. Too often projections are made for any time horizon of interest without regard to the dimensionality of the model. Sometimes single tree models have given unreasonable values after projections for 100 or 200 years, and this fact has been used to discredit the entire approach. On the other hand, stand (single equation) models cannot be used for short-term evaluations of tree spacing relationships, and this fact has been used to criticize this well-established approach. Obviously there are needs requiring growth projection systems with a spectrum of resolution levels.

In theory, it seems reasonable that one should be able to go from one resolution level to another without having to calibrate the denominator in the modifier function for every level. One would, of course, have to calibrate at several, say three, levels, but from then on one would simply input to the system the resolution level a user desires or can afford. The modifier function, as well as the other parts, would be computed by interpolation or extrapolation from the three calibrated levels.

USE ENVIRONMENTS

Once the initial calibration of the projection system has been completed there are at least three types of environments in which it may be used. One environment is where the objective is simply to project plot tree lists to a future time. This input-transformation-output schema is shown in figure 4. The algebraic forms come from the design phase, coefficients come from the calibration phase, and the input tree list may come either from temporary or permanent forest plots. No projection system should be used in this manner for prolonged periods of time without also being used in the second environment.

In the second environment accuracy and precision of the projection system are evaluated. The schema for this environment is shown in figure 5. Here, in addition to input tree list, projection system, and output tree list, I have added the "experience", detector and controller parts to the schema. This environment requires that permanent forest growth plot data be used for the input tree list and that, after a number of years has elapsed, a remeasurement of the plots be made and used as "experience". A comparison of the difference is the function of the detector, which passes the distribution of differences back to a controller, which uses it to modify the algebraic forms and coefficients in the projection system.

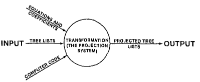
EQUATIONS AND COEFFICIENTS
INPUT
TREE LISTS
TRANSFORMATION (THE PROJECTION SYSTEM)
PROJECTED TREE LISTS
OUTPUT
COMPUTER CODE

Figure 4.—*Schema of environment for unverified projections.*

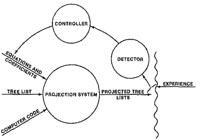

Figure 5.—*Schema showing processor in a cybernetic system involving use of permanent forest growth plots to correct the processor.*

Reason suggests that with an untested projection system one should not wait too long for another measurement — an "experience" reading. It also suggests that when little deviation is detected after such a reading one may want to project for a longer period before the next check is made with "experience". Under conditions where exogenous variables are thought important, for example, climatic variations, it may be desirable to both fix an upper limit to the projection period and automatically use the most recent "experience", in place of the most distant, to recalibrate the model. This would produce a running average type model, as opposed to a cumulative model wherein all past measurements are included in the calibration data base.

A third use environment is shown schematically in what I have called the scenario environment (fig. 6). Here, the projection system is instructed to execute various management recommendations and guides, and the outputs of each set, each scenario, are examined in light of various performance indices

SUMMARY

Forest growth and yield research appears to be entering a new era where mathematical models are evaluated for generality of applicability as well as the traditional goodness of fit of equations to data. The basic model structure discussed in this paper is shown to be adaptable to single equation models as well as to models with large numbers of simultaneous equations. By using a relativistic approach to size class assignment the approach is made more computationally efficient than when fixed-diameter intervals are used. The concept of calibrating three resolution levels of the model and computing the needed coefficients in other resolution levels is presented, as is the notion that length of projection period should determine dimensionality of the model. Variable resolution level, relativistic models will be more easily tailored to the individual needs and budgets of users than are today's models.

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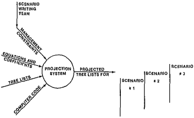

Figure 6.—*Schema of scenario environment. The growth processor is used to produce alternative output vectors for the inputs selected by the scenario writing team.*

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DATA BASE

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Principal Mensurationists

The need for growth projection systems that apply to all forest conditions of an ownership has been emphasized by Lundgren and Essex (1979), and a candidate projection system has been described by Leary (1979). Others have been described by Ek and Monserud (1974), Hegyi (1975), and Stage (1973). All the descriptions make it clear that a satisfactory model will not be developed by analyzing a few growth plots. Rather, we need to have available a large amount of data. The data base for the current study covers 44 different studies in Minnesota, Wisconsin, and Michigan. Included are 1,501 plots containing 484,574 individual tree measurements on 92,649 trees.

The purpose of this paper is to describe the data base used to calibrate the growth projection system from several perspectives: space, time, age, site, and species. The data base used to validate the model is presented in Leary *et al.* (1979) and the data needed to implement the model are escribed in Hahn and Brand (1979).

SOURCES OF DATA

Calibration of the growth projection system equires permanent growth plots with individual ree records at several points in time. Our sources were plot records from (1) cutting experiments, (2) demonstration woodlots, (3) industrial continuous forest inventory, and (4) personal records of forest growth. In each case, the growth information came from permanent sample plots, rather than complete forest enumeration. The plots were all fixed size—either circular or square. Square plots varied from 0.1 to 0.5 acre, and circular plots varied from 0.1 to 0.2 acre. In some cases only trees greater than 9 inches d.b.h. were measured; for others all trees greater than 0.6 inches d.b.h. were measured. Some plots were located randomly or systematically throughout the forest lands of an ownership. Other plots, coming largely from experimental forests, were all from similar quality sites.

Measurement data from five properties were supplied to us on magnetic tape, and data from five other properties were supplied on cards. The data from the remaining 34 properties were supplied on the original field data collection forms. Inconsistent form layout between studies added considerably to the work of preparing field sheets for direct keypunching. Data transcribing was avoided as much as possible.

During 1975 and 1976 each plot was visited and one or more of the following variables was measured or estimated: d.b.h., crown class, crown ratio, and tree status. These data were added to the already existing data.

DISTRIBUTION OF DATA SOURCES

The spatial distribution of the data sources is shown in figure 1. Lower Peninsula of Michigan data are entirely from plantations of red and jack pine. Wisconsin data are mostly from natural stands of hardwoods—oak-hickory type in the south and west, and northern hardwoods in the north—and northern white-cedar. The two Wisconsin plantation studies are Star Lake red pine adjacent to Michigan and white spruce near the Menominee Reservation. The northern Minnesota studies include natural and plantation studies of red pine and jack pine, and natural stands of quaking aspen, white pine, and black spruce.

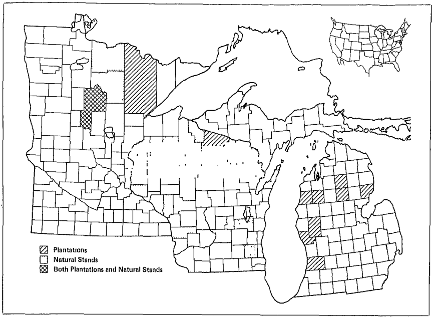


Figure 1.— *Location of counties with one or more calibration data sources in Minnesota, Wisconsin, and Michigan.*

The distribution of data sources by 1977 age (time) and site (matter/energy) is shown in table 1. Each age-site table is for a different forest type. Although the total number of plots in table 1 is 1,501, 1,529 plots were available for use; however, the variable necessary for determining forest type was not available on 28 plots. The stand age at which plot measurements were used may be as much as 20 to 30 years less than the ages given in table 1. For example, the 26 jack pine plots in the 31-40 (age)-50 (site index) cell were 36 years old in 1977. They had, however, been measured since 1953 (table 2), so the measurements began when the trees were 12 years old.

The distribution of the data sources by starting date and elapsed time between measurements is shown in table 2. The oldest measurements were made in 1938, giving nearly 40 years of observations. Others are not far behind at 30+ years of records. Not every plot on a property was measured where an "X" occurs in table 2.

PROCESSING AND STORING DATA

Information on field sheets was either keypunched directly or transcribed to 80-column sheets and then keypunched. The card data, as well as all data from magnetic tapes, were put into a standard format on disk files for editing. Following correction of all apparent data errors, the edited data were joined with the following information and placed in a master format:

(1) Mode of stand origin, treatment, replication, property number, plot number.
(2) Site index, site index species, and year of stand origin for up to five species per plot.
(3) Soils information.
(4) Forest type.
(5) Denominator of plot size fraction.
(6) Number of trees on plot.
(7) Number of measurements in time.

Table 1.— *Distribution of 1,501 permanent forest growth plots used for model calibration by age and site within 13 forest types in the Lake States (Site class 30 ranges from 25 to 34, 40 from 35 to 44, etc.)*

JACK PINE

Age class	Site index class 30	40	50	60	70	80	90	100	Total
21-30		1			1				2
31-40			26		1		1		28
41-50				1	65				66
51-60				1					1
61-70		62							62
Total									159

SWAMP CONIFER

Age class	30	40	50	60	70	80	90	100	Total
31-40					1				1
41-50			1						1
51-60		1	1						2
Total									4

TAMARACK

Age class	30	40	50	60	70	80	90	100	Total
21-30				1					1
71-80		1							1
81-90	2								2
Total									4

NORTHERN WHITE CEDAR

Age class	30	40	50	60	70	80	90	100	Total
31-40					1				1
61-70		1							1
71-80	1								1
91-100	1	20							21
100+	1								1
Total									25

OAK-HICKORY

Age class	30	40	50	60	70	80	90	100	Total
31-40					1	1			2
41-50			1		2	2			5
51-60		1		1	4	3			9
61-70					26	2			28
71-80			1		3				4
81-90		1	1	1	30				13
91-100			6	22	1				29
100+			11	40	15	1			67
Total									157

RED PINE

Age class	Site index class 30	40	50	60	70	80	90	100	Total
0-20						1			1
21-30					1	3		1	5
31-40					39	1			40
41-50				2	1				73
51-60		14	18		1				33
61-70			47		63				110
81-90			18						18
91-100				1					1
100+			149	14					163
Total									457

WHITE SPRUCE-BALSAM FIR

Age class	30	40	50	60	70	80	90	100	Total
0-20			1						1
21-30			1	1		1			3
31-40			2	6	3		1		12
41-50			1	5	2	1			9
51-60		1	1	5	11	1			19
61-70		1		21	3				25
71-80	1								1
Total									70

LOWLAND-HARDWOOD

Age class	30	40	50	60	70	80	90	100	Total
41-50				1					1
51-60	1	1							2
61-70					1				1
71-80		1							1
100+	1		2						3
Total									8

QUAKING ASPEN

Age class	30	40	50	60	70	80	90	100	Total
0-20		1	1						2
21-30		1	2	2	1	1			7
31-40		6	2	5	1		1		15
41-50		2	6	6	3	3			18
51-60		2	2	6		28	1		39
61-70		1		6	7				14
71-80			2						2
Total									97

WHITE PINE

Age class	Site index class 30	40	50	60	70	80	90	100	Total
0-20			1		1				2
21-30		1							1
31-40			1						1
41-50				4	2				6
51-60			1		1	1			3
61-70					3				3
71-80					10				10
91-100				17					17
100+				27					27
Total									70

BLACK SPRUCE

Age class	30	40	50	60	70	80	90	100	Total
21-30			1	2					3
31-40	1			1					2
41-50	12	1	3		1				17
51-60	1		1						2
61-70			2	1					3
100+		1							1
Total									28

NORTHERN HARDWOODS

Age class	30	40	50	60	70	80	90	100	Total
0-20		1							1
21-30			1	2	2		1		6
31-40		2	1	5	5				13
41-50		2	6	11	20	2			41
51-60			7	14	2	37			60
61-70	1	1	6	31	112	9			160
71-80		2	9	3	39				53
81-90		2	1	1	14				18
91-100		1	1	17	3				22
100+		2	27	4	3	4			40
Total									414

PAPER BIRCH

Age class	30	40	50	60	70	80	90	100	Total
21-30		1							1
31-40				1					1
41-50		1		2	1				4
61-70			1						1
91-100				1					1
Total									8

Table 2.—*Distribution of permanent growth plot measurements after the earliest measurement for calibration data base from Michigan, Wisconsin, and Minnesota.*

PROPERTY/STUDY	MAJOR SPECIES	YEAR OF EARLIEST MEASUREMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
MICHIGAN																																						
NC-40 (Wellston)	Jack pine	53	X					X	X	X	X	X				X				X	X			X	X													
NC-178 (Crawford Co.)	Jack pine	60			X	X												X																				
NC-431 (Bosom Field)	Red pine	52			X	X	X			X	X	X				X	X				X	X	X	X		X	X											
NC-433 (Buck Creek)	Red pine	51			X	X	X		X	X	X	X			X	X	X			X	X	X			X	X	X											
NC-434 (Croton Dam)	Red pine	52				X			X		X					X					X					X												
NC-436 (Ravenna)	Red pine	60				X	X		X	X						X	X																					
NC-437 (Sooner)	Red pine	60		X			X										X																					
WISCONSIN																																						
Apple River THF	Oak, White pine	47						X		X		X				X													X									
Argonne THF	Northern hardwoods	47	X	X	X	X	X	X	X	X			X				X	X				X	X					X										
Cairns THF	Northern hardwoods	49				X				X				X					X								X											
Ellsworth THF	Northern hardwoods	48					X						X		X					X								X										
Chilsen THF	Hemlock, Yellow birch, Maple	47		X	X	X	X		X		X			X				X	X	X								X										
Dundee THF	Red oaks, Northern hardwoods	47			X				X			X					X												X									
Hardies Creek THF	Red oak, White oak	51	X						X			X													X													
Brenholdt THF	White pine	52							X		X													X														
Waussaukee THF	White & Red pine, Aspen	47					X		X				X				X							X					X									
Champion Valley THF	Northern hardwoods	47								X			X							X									X									
Star Lake	Red pine	43					X						X							X								X						X				
Butler's Woods	Oak, Basswood, Maple	45	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X											X							
Nettleton	Northern hardwoods	38		X				X		X	X	X					X	X			X		X				X	X					X					X
Stone's Woods	Oak, Hickory, Maple	45	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X											X							
Shaugnessey's Woods	Oak, Hickory, Maple	45	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X											X							
NC-26	Balsam fir, Aspen	49					X	X				X		X				X		X		X	X	X	X		X											
NH-25	Northern hardwoods	51						X				X					X					X			X													
NH-138 (Argonne)	Northern hardwoods	56						X		X				X			X			X																		
NH-138 (Medford)	Northern hardwoods	58				X			X				X					X																				
NC-18	White spruce	49						X	X								X										X											
NC-32	N. White cedar	46										X												X						X								
Owens-Illinois	Northern hardwoods Conifers	64					X					X																										
MINNESOTA																																						
NC-56 (Pike Bay)	White pine	54	X				X					X					X					X																
NC-57 (Pike Bay)	Jack pine	53				X	X					X				X						X																
NC-59 (Little Fork)	Black spruce	54													X							X																
NC-76 (Cutfoot)	Red pine	49				X	X				X	X			X		X			X		X			X		X											
NC-78 (Cutfoot)	Red pine	50					X					X							X			X			X			X										
NC-79 (Bena)	Red pine	49					X					X					X					X					X											
NC-83 (Marcell-Clubhouse)	Red pine	44								X					X		X				X				X													
NC-84 (Lake 13)	Red pine	44												X	X	X	X				X	X				X	X				X	X						
NC-96 (Pike Bay)	Aspen	56					X				X					X					X																	
NC-98 (Aurora)	Jack pine	68					X																															
NC-142 (Portage Lake)	Red pine	47									X				X					X						X				X								
NC-155 (Birch Lake)	Red pine	57					X					X					X				X																	
NC-163 (Mando)	Red pine	57				X					X					X					X																	

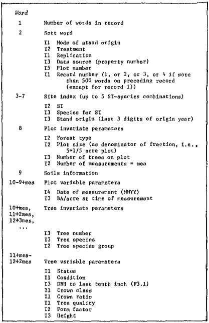
[1] *An "X" denotes that at least one plot on the property was measured at the number of years after the earliest measurement given by the column label. Only Butler's, Stone's, and Shaugnessey's Woods were measured in their entirety every year.*

(8) Dates (month and year) of each measurement.

The master format consists of a maximum of 512 word blocks of binary information. The block length is specified because of input/output buffer length on the Control Data Corporation's Cyber '74 computer. Each word can contain up to 14 integer values to code information. The master format and contents of each word are given in figure 2. An example output of a permanent growth plot record is shown in figure 3. Specific objectives of the master format were to (1) maximize efficiency in reading/writing and storing the master files, and (2) have information in a standardized format because five or six different persons may be getting information from the master at any one time.

RETRIEVAL OF INFORMATION FROM DATA BASE

Because the information is written in blocked nary form with 14-character words, one must e simple FORTRAN statements to extract the sired values from a word in a block. For exam- e, to analyze mortality, the master file is read nd the desired information extracted to estab- lish an analysis-specific file. Most of the sort word contents are normally carried into the analysis-specific files. This information, together with other information, aids in tracing any problem observations and for organizing analyses by state, species, mode of stand origin, etc. The analyses then proceed species-by- species for the 26 species groups for which functions are developed.



```
  Word

    1        Number of words in record

    2        Sort word

             I1  Mode of stand origin
             I2  Treatment
             I1  Replication
             I3  Data source (property number)
             I5  Plot number
             I1  Record number (1, or 2, or 3, or 4 if more
                    than 500 words on preceding record
                    (except for record 1))

   3-7       Site index (up to 5 SI-species combinations)

             I2  SI
             I3  Species for SI
             I3  Stand origin (last 3 digits of origin year)

    8        Plot invariate parameters

             I2  Forest type
             I2  Plot size (as denominator of fraction, i.e.,
                    5=1/5 acre plot)
             I3  Number of trees on plot
             I2  Number of measurements = mea

    9        Soils information

 10-9+mea    Plot variable parameters

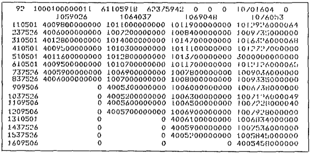
             I4  Date of measurement (MMYY)
             I3  BA/acre at time of measurement

 10+mes,     Tree invariate parameters
 11+2mes,
 12+3mes,
    ...
             I3  Tree number
             I3  Tree species
             I2  Tree species group

 11+mea-
 12+2mea     Tree variable parameters

             I1  Status
             I1  Condition
             I3  DBH to last tenth inch (F3.1)
             I1  Crown class
             I1  Crown ratio
             I1  Tree quality
             I2  Form factor
             I3  Height
```

Figure 2.— *Lake States FREP binary master record format.*



```
  92  1000100000011  61105918  62375942  0   0   0   10701604  0
              1059026         1064037        1069048        1076053
  110501 400980000000 101100000000 101190000000 101292600064
  237526 400600000000 100720000000 100840000000 100973500000
  310501 401280000000 101400000000 101470000000 101632600068
  410501 400950000000 101030000000 101110000000 101272700000
  510501 401160000000 101280000000 101370000000 300000000000
  610501 400950000000 101070000000 101170000000 101212600065
  737526 400590000000 100690000000 100780000000 100903600000
  837526 400600000000 100700000000 100800000000 100933500000
  909506            0 400530000000 100600000000 100673800000
 1037526            0 400520000000 100630000000 100712600049
 1109506            0 400560000000 100650000000 100722800040
 1209506            0 400570000000 100690000000 100792800000
 1310501            0            0 400610000000 100683400000
 1437526            0            0 400590000000 100753600000
 1537526            0            0 400520000000 100584500000
 1609506            0            0            0 400545800000
```

Figure 3.— *Sample output listing of a single plot from the master file. Contents and meaning of each word are given in the master format in figure 2.*

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POTENTIAL DIAMETER GROWTH FUNCTIONS

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Principal Mensurationists

When seeking an unknown value of a numerical variable it is common practice to bracket the unknown by two values—one greater than the unknown and the other less. Mathematicians have used this practice for many years to find the solution of nonlinear algebraic equations. A simple example is the problem of finding the value of x that satisfies the equation $x^2 = y$, when $y = 142$.

The square root of a number can be estimated by guessing a number such that its square is greater than y and then guessing another with square less than y; e.g., $12^2 = 144$ and $11^2 = 121$. The desired value is then known to be between 12 and 11. By judicious guessing one quickly gets several correct digits in the value of x.

The idea of establishing an upper and a lower bound to the rate of a biological process is a very useful modeling device. In the case of the accretion processes it is reasonable to take zero as the numerical value of the lower bound. This paper is concerned with estimating the largest possible numerical values—the potential—for increase in diameter at breast height (d.b.h.) by all major forest species in the Lake States. Our concern is with both the potential diameter growth rates of individual trees and the diameter growth potential of all trees in a stand.

The concept of a growth potential has been used in mathematical modeling of biological processes for many years (Verhulst 1965, Lotka 1956, Volterra 1931, Gause 1969). The potential is given by r in the generalized differential equation of population growth: $\frac{dY}{dt} = r\,f(Y)$.
The term r refers to the rate of self-renewal of the population in question. Its effect dominates the right-hand side of the equation as long as resources are not limiting.

Foresters have only recently begun using the concept of potential in forest growth models. Arney (1974), Hegyi (1974), Ek and Monserud (1974) have used potential height growth as the upper limit of growth. Potential height growth is usually estimated by taking the first forward difference, with respect to age, of equations to predict height based on site index and age (see Lundgren and Dolid 1970). The use of breast height diameter growth to characterize a tree's potential is less common than the use of height (Newnham 1964, Chen 1976). A disadvantage of this approach is that there are no standard functions relating tree and environment variables to diameter growth, as there are to relate tree and environmental variables to tree height.

In theory, the following tree and environment variables affect the amount of lateral stem growth at breast height: (1) current d.b.h., (2) amount of foliage, (3) respiring surface area, (4) distance from breast height to center of live crown, and (5) abiotic flux as reflected by site index. The relation between each of these variables and diameter growth was extracted from the literature by Brand:[1]

(1) The closer the crown to breast height, the more stem growth (Duff and Nolan 1953).

(2) Less foliage is needed on a good site (Assmann 1970).

(3) Amount of foliage is greater on a good site (Long and Turner 1975).

(4) Respiring surface area is related allometrically to d.b.h. (Pienaar and Turnbull 1973).

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[1] *Brand, G. 1976. Personal communication, St. Paul, Minnesota.*

There are many ways of combining variables used to characterize each of these relations. Two broad categories are strictly additive association and strictly multiplicative association. Of course, one may have combined additive and multiplicative.

APPROACH

Our objective was to develop species-specific diameter growth equations that characterize the potential of Lake States tree species to increase in d.b.h. We have used a combined additive-multiplicative model that requires knowledge of the following variables: d.b.h., crown class, crown ratio, and site index (age 50 base). Crown ratio is the ratio of full live crown length to total tree height.

Data Base

Individual tree diameter measurements at two points in time were collected on dominant and codominant trees of 26 species in Michigan, Wisconsin and Minnesota. A total of 51,149 trees came from permanent growth plots or from forest survey plots in Wisconsin; of these, 36,726 were high-quality dominant and codominant trees usable for estimating potential growth.

All trees had crown class and crown ratio estimates at the last measurement. Plot site index was also estimated. The geographical distribution of plots, and the distribution in the age-site matrix is given in Christensen *et al.* (1979).

The diameter growth (ΔD) for each tree was determined over a period approximating 10 years. This was the dependent variable in a nonlinear regression analysis. The independent variables were initial diameter (D), final crown ratio (CR), and plot site index (SI). These data for all dominants and codominants were grouped by species, as shown in the tabulation above, by 1-inch diameter classes, by 10-foot site index classes, and by 10-percent crown ratio classes. If a cell contained just one tree, it was included in the most appropriate adjacent class. For each cell the means were computed for each variable, i.e., $\overline{\Delta D}, \overline{D}, \overline{SI}, \overline{CR}$, as well as the standard deviation of diameter growth, $s_{\Delta D}$.

Species	Number of trees
Jack pine	4,625
Red pine	11,663
White pine	845
White spruce	1,772
Balsam fir	1,277
Black spruce	1,006
Tamarack	165
Northern white-cedar	1,070
Hemlock	140
Black ash	240
Cottonwood Silver maple Red maple	937
Elm	898
Yellow birch	343
Basswood	1,092
Sugar maple	3,753
White ash	866
White oak	517
Select red oak	1,555
Other red oak	290
Hickory	349
Bigtooth aspen	261
Quaking aspen	2,124
Paper birch	748
Other hardwoods	190
Total number of trees	36,726

The mean diameter growth, $\overline{\Delta D}$, for each cell applies to dominants and codominants. Since it was our desire to estimate the potential of trees to increase in diameter at breast height, it was assumed that the diameter growth values were normally distributed around this mean (see fig. 1). The potential is, of course, greater than the average. We took the potential to be the mean diameter growth plus 1.65 standard deviations ($\Delta D+1.65s_{\Delta D}$). This diameter growth corresponds to the 95th percentile of dominant and codominant diameter growth.

Mathematical Model

The following four mathematical models were examined. All were nonlinear in form and combined additive-multiplicative with a core Richards-type function that has anabolic and catabolic terms (Richards 1959):

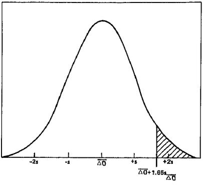

Figure 1.—*The assumed normal distribution of diameter growth of dominant and codominant trees for a 10-foot site index class and 10-percent crown ratio class. The shaded area represents the fastest-growing 5 percent of the trees.*

$$\frac{\Delta D}{\Delta t} = b_1 SI + b_2 D^{b_3} + b_4 CR\ D \qquad (1)$$

$$\frac{\Delta D}{\Delta t} = b_1 CR + b_2 D^{b_3} + b_4 SI\ D \qquad (2)$$

$$\frac{\Delta D}{\Delta t} = b_1 + b_2 D^{b_3} + b_4 SI\ CR\ D \qquad (3)$$

$$\frac{\Delta D}{\Delta t} = b_1 + b_2 D^{b_3} + b_4 SI\ CR\ D^{b_5} \qquad (4)$$

The pattern of each is $\frac{\Delta D}{\Delta t}$= ["intercept" term] + [catabolic term] + [anabolic term]. The signs on b_1, b_3, b_4, and b_5 are positive, and b_2 is negative. The "intercept" term gives the growth approached as d.b.h. approaches zero; in equations (3) and (4) $\lim_{D\to 0} \frac{\Delta D}{\Delta t} = b_1$.

This limit is a function of SI in (1) and of CR in (2). The catabolic term, $b_2 D^{b_3}$, has the form of an allometric relation, as required of a relation characterizing respiring surface area (Pienaar and Turnbull 1973). Several anabolic terms were tested. The terms in equations (3) and (4) make the most sense, because total tree height is multiplied by crown ratio, giving crown length. Stem diameter has often been related to crown diameter. The product of crown length and width gives an approximation, uncorrected for crown shape, of crown volume.

Fitting Procedures

Each of the above models was fit to both mean and potential ΔD values for each cell of each species using Marquardt's algorithm modified for weighted nonlinear least squares regression (Marquardt 1963). Model evaluation criteria were: qualitative behavior of the solution to each equation using the estimated parameters b_i; pattern formed by residuals (predicted minus observed diameter growth) plotted against initial diameter, crown ratio, and site index; and goodness of fit as measured by R^2. For some species the qualitative behavior of the solution was unrealistic because an insufficient number of observations was available at large diameters. To compensate, the silviculture literature was consulted and from it we determined the upper limit of d.b.h. for the species. A very small ΔD value was assigned to the diameter, an average crown ratio and site index were attached, and the "add" values were inserted into the data base as needed to insure more realistic behavior of the function at large diameters.

Results

Models (3) and (4) gave better results than (1) or (2) as judged by model selection criteria. The results of the regression analyses of models (3) and (4) for 26 species for both average growth and potential growth were examined and one or the other was selected as the best model. The coefficients, number of cells, and R^2 values are given in the tabulation. When the b_5 parameter is exactly 1.0, the model selected was number (3). Otherwise, model (4) was selected. Figure 2 shows potential diameter growth values for species commonly found in the pine type: red pine, white pine, jack pine, and quaking aspen. The crown ratio was set at 4.0 and site index at 65.

DISCUSSION

To use the potential diameter growth function it is necessary to know the initial diameter, crown ratio, and site index. Crown ratio is not a commonly collected tree variable, but the average

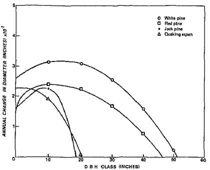

Figure 2.—*Predicted potential diameter growth values for species commonly found in the pine type in the Lake States. Site index is 65 and crown ratio is 4.0 for each species.*

crown ratio for a stand can be estimated by knowing mean stand basal area, mean diameter, and stand age (Holdaway *et al.* 1979).

The inferential population for equations (3) and (4), with coefficients in table 1, is the natural stands and plantations of the 26 species in Michigan, Minnesota, and Wisconsin. No attempt was made to separate trees in plantations from those in natural stands, because the individual tree is not thought to behave differently.

The approach used for potential diameter growth is somewhat analogous to that used for models based on potential height growth. The latter use height-age relations for dominants (site index curves). We used diameter growth-diameter relations for the fastest-growing dominants and codominants. By taking the 95th percentile of diameter growth, we intended to approximate diameter growth rates of forest trees not subject to significant competition from neighboring trees. We judged open-grown trees as not suitable for potential diameter growth studies because of the significant difference in the distribution of increment among bole, branches, and limbs.

Table 1.—*Number of cells and coefficients for average and potential diameter growth functions for 26 species in the Lake States*[1]

	Number of	Average						Potential					
Species group	groups	b_1	b_2	b_3	b_4	b_5	R^2	b_1	b_2	b_3	b_4	b_5	R^2
1 Jack pine	126	0.09349	−0.00005	2.9144	0 00003	1.0	0 44	0.16062	−0 000009	3 6245	0 00004	1.0	0 35
2 Red pine	282	.05309	− 00013	2 0468	.000167	38337	.64	09446	−0.00012	2.0598	00035	.24225	54
3 White pine	181	15242	−.00030	1 9860	.00003	1.0	.34	25576	−.00088	1.7263	00004	1.0	28
4 White spruce	72	18769	− 00776	1 3904	.00006	1.0	.51	.17056	− 014516	1 0660	.00052	27298	74
5 Balsam fir	121	037102	−.00028	1 9768	.00016	37750	.64	12200	−.00080	1 9890	.00006	1.0	.50
0 Black spruce	105	.06667	− 00049	2 1592	00003	.95715	.26	.10713	−.00107	2 0017	.00006	61127	.39
7 Tamarack	55	05279	−.000002	3 4307	.00003	1 0	.29	11147	−.00001	3 0685	00003	1 0	22
8 N white-cedar	93	.06480	− 00000002	4.7486	00012	15411	41	.13403	−.000001	3.6880	00002	1.0	17
9 Hemlock	69	.11278	−.000000009	4.3699	000007	1.0	25	16872	−.0000003	3 5738	00001	1.0	25
11 Black ash	84	.035027	− 000000001	5 3434	.00010	46385	.41	058607	−.000000006	5 0559	.00024	31553	.26
12 Cottonwood													
13 Silver maple	175	.06793	− 00038	1.5944	00020	.29971	.44	.10946	− 00004	2 2226	.00050	06626	30
14 Red maple													
15 Elm	179	14862	−.00016	1.9910	.00002	1.0	.12	.28496	−.00182	1.5297	00003	1.0	08
16 Yellow birch	96	11333	− 00005	2.3334	000006	1 0112	.12	.15155	−.000003	3.3104	00007	57302	.18
17 Basswood	169	.16782	− 00025	1.8136	000008	1.0	16	.25402	−.000004	2.9396	00001	1.0	06
18 Sugar maple	272	.13309	−.00051	1 7002	.00002	1 0	29	.18772	−.000007	2 5639	.00028	06385	.33
19 White ash	139	.10829	−.00025	2.0926	.00003	1.0000	.36	21167	−.000003	3 3131	.00001	12430	.27
20 White oak	152	08114	− 0000004	3.1795	00004	.51666	.26	.12654	− 000004	2.7538	00005	.82086	21
21 Select red oak	269	098793	− 0000002	3.3873	00008	.41155	.36	.15535	− 00000010	3 5367	00018	25900	22
22 Other red oak	62	10142	− 0000002	3.7728	00012	.20604	.46	17358	− 00007	2.4451	00003	.96222	30
23 Hickory	93	.071908	−.000008	2 7684	.00003	69378	44	.16471	−.02161	1 3949	00004	.71628	.26
24 Bigtooth aspen	79	.14120	−.00029	2.1217	.00003	1 0	.28	.23490	−.00942	1 1641	00036	.16386	.33
25 Quaking aspen	201	.11789	−.00015	2 3618	.000042	1.0	.39	.21645	−.000091	2 6030	.000044	1.0	.31
26 Paper birch	112	.07565	−.00059	2.0175	.00005	1 0003	.48	10971	−.00032	2 0236	.00034	.21286	.41
30 Other hardwoods	76	.19662	−.00954	1 0500	.00002	1.0	.44	.37510	− 02479	6.6288	00004	1.0	.35
Total	3,284												

[1] Crown ratio variable used is 3.0 rather than 0.3. General model is $\frac{\Delta D}{\Delta t} = b_1 + b_2 D^{b_3} + b_4 SI\ CR\ D^{b_5}$.

To estimate stand potential growth, the mean stand d.b.h., mean crown ratio (either observed or predicted), and site index are substituted into equation 4, along with the appropriate coefficients from table 1; the potential growth of a single tree is then computed. This is the estimated rate of growth of a free-to-grow tree. Potential stand growth is simply the number of trees in the stand times the free-to-grow rate.

CONCLUSION

Potential and average diameter growth rates for 26 species in the Lake States are adequately predicted by combining initial tree diameter, crown ratio, and site index, into a nonlinear model containing three terms: intercept, catabolic, and anabolic. The intercept term is a species-specific constant, the catabolic is an allometric relation of tree diameter, and the anabolic term is a product of crown length and an allometric relation of tree diameter. Since the potential diameter growth function characterizes free-to-grow conditions, potential stand diameter growth is simply the product of tree potential and number of trees.

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ESTIMATING MEAN STAND CROWN RATIO FROM STAND VARIABLES

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The amount of foliage in a stand of trees is an important indicator of the growth one may expect from the stand. Although this relation has been known for decades, the amount of foliage is not an easily determined variable that can be used in equations to predict stand growth. Field measurements on permanent growth plots have traditionally concentrated on stem dimensions (i.e., diameter and height), with supplemental correlation studies to associate crown diameter with stem diameter and other stand variables. An exception is the national forest survey, which has included measurements on tree crown ratio for many years. Recall that crown ratio is the fraction of total tree height in full live crown. Throughout this paper we express the "fraction" as an integer between 0 and 10 — i.e., a crown ratio of 2 means a tree has 11-20 percent of total height in full live crown.

Crown ratio was found to be an important variable for predicting tree growth (Hahn and Leary 1979), and was incorporated into an equation to predict potential diameter growth in a stand. Because the potential growth equations require mean stand crown ratio as an independent variable, and some prospective users of the generalized growth projection system (Hahn and Brand 1979) may not have included this variable in their plot data, it is necessary to develop equations to approximate mean stand crown ratio.

MODEL RATIONALE

From a theoretical analysis it is suggested that the mathematical model should exhibit the following qualitative properties:

(1) solving the mathematical equation must give a value between 0 and 10,

(2) crown ratio must not increase as BA (basal area per acre) increases,

(3) crown ratio must ultimately decrease as age or average diameter increases, and

(4) the rate of change of crown ratio with basal area must decrease as age or average diameter increases.

The exact relations, rate coefficients, etc. appear to be species-specific.

The core model tried initially was:

$$\overline{CR} = \frac{b_1}{1 + b_2\, BA} \tag{1}$$

Although this model satisfied condition (2) above, it did not satisfy (1) and, because only basal area is used, it did not satisfy the important conditions in (3) and (4). The additional variables of age and average diameter were used with equation (1) to produce models satisfying conditions (1) through (4).

METHODS

The initial data base (Christensen *et al.* 1979) consisted of 1,529 permanent growth plots in the Lake States. Based on methods of treating plots of mixed species (Leary and Holdaway 1979), each plot was assigned as a pure species plot or as a combination of up to three species. In this study only plots fitted as "pure" or as containing two species were used. The individual tree crown ratios, measured on each plot in the last measurement year, were averaged to obtain a species mean crown ratio $(\overline{CR})$. Plot basal area was averaged over the 10 years preceding the crown ratio measurement. When no measurement had been taken

10 years preceding, the measurement nearest to 10 years was used. Only plots with no major disturbances (i.e., losses to mortality and/or cutting) during the interval were included. If the final basal area was less than 90 percent of the initial basal area, a major disturbance was assumed to have occurred and the plot was not used. For all acceptable plots mean stand diameter (AD) was calculated at the final measurement year and the stand age (AGE) was calculated at the midpoint of the interval.

RESULTS OF ANALYSIS

Because of insufficient or poorly distributed data for some species, it was necessary to group species as follows:

Group number	*Species group*
1	jack pine
2	red pine
3	white pine
4	white spruce, balsam fir
5	black spruce, tamarack
6	northern white-cedar, hemlock
7	cottonwood, silver maple, red maple
8	black ash, white ash, elm, basswood
9	yellow birch
10	sugar maple
11	white oaks, select red oaks, other red oaks, hickory
12	bigtooth aspen, quaking aspen, paper birch

The analysis showed two patterns of relations — one for conifers and another for hardwoods. The conifers tended to have much larger crown ratios for low basal areas than did the hardwoods. Mean plot crown ratios for conifers were seldom greater than 8 or less than 2. The range for hardwoods was much smaller; crown ratios seldom exceeded 5 even at very low basal areas. Fourteen different models, each containing the core relation, were fit to the data from 12 species groups using nonlinear regression methods (Marquardt 1963). The following model was selected for conifers:

$$\overline{CR}=\frac{8e^{b_1 AGE}+2}{1+\frac{b_2}{AD}BA}\text{, where} \qquad (2)$$

AD=arithmetic mean d.b.h.,

BA=basal area in square feet,

AGE=years from stand establishment, and

b_1, b_2=unknown numerical constants.

Equation (2) coefficients for each conifer species group are as follows:

Species group	b_1	b_2	R^2	*Standard error of estimate*	*Number of plots*
Jack pine	−0.0187	0 0849	0 38	3 5	123
Red pine	−.01270	.0504	.42	4.3	384
White pine	− 0060	1399	42	2 1	62
White spruce-balsam fir	−.0058	.0815	71	3 7	52
Black spruce-tamarack	−.0039	.1045	.12	7 0	41
Northern white-cedar-hemlock	−.0213	.0055	.41	3 7	42

The amount and rate of change of crown ratio with age, average diameter, and basal area for hardwoods was quite different from that for conifers. In general, hardwood crown ratio changed very little over the range of stocking, size, and age available in the data base. The model for hardwoods was established as the following variation of the core model:

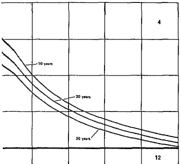
$$\overline{CR}=b_1\,AGE+\frac{b_2}{1+\frac{b_3}{AD}BA}. \qquad (3)$$

Coefficients in equation (3) for the hardwood species groups are:

Species group	b_1	b_2	b_3	R^2	*Standard error of estimate*	*Number of plots*
Cottonwood, silver maple, red maple	−0 0186	6.910	0.0476	0 46	2.2	27
Black ash, elm, basswood, white ash	0000	3 04	.0000	NA	1 4	64
Yellow birch	− 0222	6.019	.0264	.37	2 6	16
Sugar maple	−.0064	4 707	.0282	.26	1 9	143
White oaks, select red oaks, other red oaks, hickory	− 0080	4.728	.0381	.32	1.2	112
Bigtooth aspen, quaking aspen, paper birch	−.0070	4.298	.0230	.28	2.3	112

The crown ratio values for the 64 plots of black ash, elm, basswood and white ash showed so little variation that the mean value was assigned to the

ent b_2 (equation 3), and b_1 and b_3 were set at
omparing equations (2) and (3) we see that
ents b_2 (equation 2) and b_3 (equation 3) op-
o achieve the same effect: reducing the
crown ratio as basal area increases. The
e rate of change for conifers is twice that for
ods. Another major difference between spe-
ups is based on the "intercept" values — i.e.,
wn ratio as basal area, age, and average
er approach zero. For conifers, the "inter-
s 10, a full live crown. For hardwoods the
pt is b_2, and ranges from 6.9 for cottonwood,
ple, and silver maple to 3.04 for black ash,
sswood, and white ash. Mean stand crown
over a range of average diameters, basal
and ages are shown for red pine and sugar
in figures 1 and 2.



1.—*Mean stand crown ratio for red pine*
s of selected ages and average diameters of
and 20 inches.

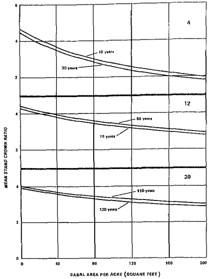


Figure 2.—*Mean stand crown ratio for sugar ma*
ple stands of selected ages and mean diameters o,
4, 12, and 20 inches.

DISCUSSION

Adequate estimates of mean crown ratio for
stand can be made if one knows the approxima
mean basal area per acre over the last 10 years, th
mean stand diameter, and the stand age. Bette
estimates can be made if one knows, say, the ru
ning average 10-year basal area. Simple poi
estimates of stand basal area will produce p
crown ratio estimates if one knows nothing ab
prior stand disturbances. A stand disturbanc
less of a problem if the stand is old or the tr
large. For example, if a red pine stand with
average d.b.h. of 6 inches, age of 40 years, a
basal area of 150 square feet is cut back to 1
square feet with no change in average stand diam
eter, the predicted mean crown ratio would in
crease from 0.30 to 0.37. However, if the mean
diameter had been 28 inches, the age 200, and the

basal area 150 square feet, cutting back to 100 square feet with no change in average stand diameter would have had no significant effect on the mean crown ratio. When applied in the projection program (Hahn and Brand 1979), the running average basal area for the past 10 years is used instead of an average of only two or three values.

If cutting or mortality changes average stand diameter as well as stand basal area, the following relation holds for the models: a decrease in diameter as well as basal area (e.g., thinning from above) results in a lower crown ratio, whereas an increase in diameter and a decrease in basal area (e.g., thinning from below) results in a higher crown ratio. This relation is logical since larger trees of the same species in a stand (dominanats and codominants) tend to have higher crown ratios than smaller trees (intermediates and suppressed).

A generalized relation between mean crown ratio, age, mean d.b.h., and basal area, applicable to both conifers and hardwoods, is given in equation (4):

$$\overline{CR} = b_1\,AGE + \frac{b_2 e^{b_3\,AGE} + b_4}{1 + \frac{b_5}{AD}\,BA}. \qquad (4)$$

For conifers, b_1 is set at zero; for hardwoods b_2 is set at zero.

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MODIFIER FUNCTION

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and **Margaret R. Holdaway**, *Mathematical Statistician*

nise in the design logic for this gen-growth projection system is that the aggregate can be represented by a combination of a potential growth of the potential. The modifier takes mathematical function since another t the amount the potential is modi-ı the amount of standing crop pres-oses of this paper are to describe results of studies to determine how ɜ interact within and between spe-

e modifier function has in the basic m is shown in equation (1):

$$\begin{bmatrix}\text{ial}\\ \text{ter}\\ \text{th}\end{bmatrix}\begin{bmatrix}\text{Number}\\ \text{of}\\ \text{trees}\end{bmatrix}[\text{Modifier}]. \qquad (1)$$

n the analysis we assume that the ıeter growth function is known, difier function as the only function numerical constants. The relation-lifier function to the data base and ıs is shown in figure 1. Because the ion is assumed known, estimation cannot begin until it has been com-hown connected to the potential crown ratio function. Under ideal here would be no need for a crown in the calibration phase because ıld have been observed at all mea-ıce actual observations could be ıdy crown ratio was estimated only ısurement, in 1975 *or* 1976; thus, if nt interval selected for estimating ıs from a prior time, it was neces-e the crown ratio used in the po-ı. Because the modifier function pend on an estimated potential ı turn may depend on an estimated crown ratio, the modifier function's sensitivity to variations in stand conditions may be less than it could be.

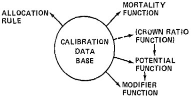

Figure 1. —*Relation of modifier function to data base, potential function, and crown ratio function. The latter is shown with parentheses and dashed line because when the crown ratio has been recently observed it is not necessary to use the crown ratio function in estimating numerical constants in the modifier function.*

REQUIREMENTS OF THE DATA BASE

Unlike the potential and mortality functions, the modifier function is based on information about all the trees on a plot. Thus, the data requirements relate to plot attributes rather than individual tree attributes. For a plot to be useful for estimating modifier function coefficients it must:

(1) have been measured three or more times,
(2) have been measured over approximately 10 years,
(3) exceed a minimum number of trees,
(4) exceed a minimum basal area density,

(5) be either a pure stand or a two-species mixture, and

(6) have not had excessive cutting or mortality in the measurement period.

The exact cutoff values for these six criteria are given in the methods section. Many plots were not used to estimate the modifier because they failed to meet one or more of the criteria.

METHODS USED TO SUMMARIZE, STORE, AND RETRIEVE DATA BASE INFORMATION

Before plot data are ready for analysis, the analyst must:

(1) Take the data, as found in the data base, on the species present on each plot and transform them into a condensed form, recording the transformation.

(2) Use this transformation information to consolidate all species present into a maximum of four species categories and then summarize the needed plot information into one physical record to enable the mass of data to be efficiently stored on disk files.

Design a system that enters a large file of information produced by step 2 (above) and picks out only those plots that match a specified species combination, "unpeels" the data, reorders the species if necessary, and lists the unpeeled data on a new file in a format ready for analysis.

Step 1 (above) needs to be discussed in more letail. We found in a preliminary study that the three needed measurements could be taken over as short an interval as 4 years without affecting the results. Whenever possible, however, we tried to have at least 6 to 8 years in the interval from first to third measurement. A computer program was written to locate where the cuts, mortality, and ingrowth occurred. From these results we manually selected an interval of at least 6 years with a minimum of disturbance due to death, cutting, or ingrowth, and a maximum number of live trees. In accommodating these constraints, the three-measurement interval tended to be as short as possible. This enabled us to use more than one interval from plots with five or more measurements. Using the chosen interval(s) we determined the percent basal area for each species on the plot, ordered the species by basal area, retained the first three species, and combined the remaining species into a miscellaneous category.

It was still necessary to reduce these species down to the principal ones. To accomplish this, a predominant species composition fitting rule was developed. The rule works approximately as follows for a plot with only two species: the plot is located in a coordinate system at the point (x_1,x_2), where x_i is the percent of plot basal area in species i (fig. 2). If the plot were a pure stand of species 1, the point would be located at coordinates (1,0); if it were a pure stand of species 2, it would be located at coordinates (0,1). All other mixtures would be located on a line between points (1,0) and (0,1) such that $x_1+x_2=1$. For a two-species plot, we calculate the distance from (x_1,x_2) to three points, (1,0), (1/2,1/2), (0,1), and assign the plot to that species combination (pure species 1, two-species mixture, pure species 2) for which the distance is smallest.

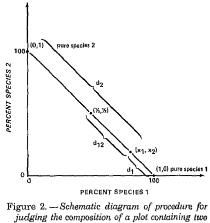

Figure 2. —*Schematic diagram of procedure for judging the composition of a plot containing two species. In this example* d_1 *is less than* d_{12}, *hence the plot will be considered a pure stand of species 1. Analogous methods are used in 3 and 4 dimensions for 3 and 4 species mixtures.*

In our work it was possible to have a maximum of four species categories. Since the species are put in descending order by percent basal area, the reference points for determining species fits are

(1,0,0,0)	corresponding to	pure species 1
(1/2,1/2,0,0)		two-species mixture
(1/3,1/3,1/3,0)		three-species mixture
(1/4,1/4,1/4,1/4)		four-species mixture

The distance from the point (x_1,x_2,x_3,x_4) to each of the four points was calculated and the species combination selected that had a minimum distance.

There was still a need to handle minor species dropped by the fitting rule. That is, something had to be done to species 4 if the fitting rule showed that the point (x_1,x_2,x_3,x_4) was closer to (1/3,1/3,1/3,0) than it was to (1/4,1/4,1/4,1/4). These species were added to a similar species based on a priority joining rule. Finally, for each plot we recorded the transformation that indicated the predominant species and what to do with all other species. For the purposes of this study only plots fit with one or two species were used and the others were skipped.

ANALYSIS METHODS AND RESULTS

Pure Stands

Once the plot information was summarized and stored in a retrievable manner, the pure stand analysis proceeded as follows: for a particular species, e.g., jack pine, the summarized data were used as boundary conditions in the nonlinear multipoint boundary value problem (Leary 1970, Leary and Skog 1972). Solve

$$\frac{\Delta Y}{\Delta t} = \text{[Potential diameter growth/tree] [NT]} \left[1 - e^{-\beta/BA}\right] \quad (2)$$

subject to the boundary conditions

$Y(t_1) = k_1$, $Y(t_2) = k_2$, $Y(t_3) = k_3$, where

Y is the sum of tree diameters,

NT is the number of trees,

BA is the stand basal area $\left(BA = .005454 \left(\frac{Y}{NT}\right)^2\right)$,

β is an unknown numerical constant (with units of basal area),

k_1,k_2,k_3 are the boundary conditions, and

$$\frac{\Delta Y}{\Delta t} = \frac{Y(t+1) - Y(t)}{(t+1) - t} \quad \text{(first forward difference).}$$

Other algebraic forms for the modifier were tested, this one most thoroughly

$$\frac{\alpha}{1 + \gamma BA}, \quad (3)$$

but were found to be less desirable based on their pattern of residuals.

Calculation of the potential diameter growth part of equation (1) requires use of mean stand crown ratio (Hahn and Leary 1979). Individual tree crown ratios on each plot had been recorded at the last measurement, but frequently the last measurement was not one of the times used to form the measurement set. Although most cases showed little difference between the β values resulting from observed or predicted crown ratios, we used only observed crown ratios whenever there was a sufficient number of interval sets. This practice resulted in a large reduction in the sets of measurements available for use. For example, red pine had 503 sets of measurements spanning three measurements, but only 241 included the last measurement and an observed crown ratio. Because we felt the observed crown ratio information was more precise, we used it unless doing so would have resulted in little or no information for a species.

The β values estimated for each measurement set judged to be pure were plotted against the mean stand diameter and their variation was studied. Based on these results, all measurement sets with 40 square feet or less of basal area stocking and with 50 percent (by number of trees) or more mortality and/or cutting between the beginning and final measurement were removed. Analysis showed the β values were independent of the number of trees, the site index, the interval length (5 to 29 years), and stand age to midpoint of the interval. There appeared to be a possible slight relationship between β and average summer temperature over the interval, but data were insufficient to warrant further study. The same held true for the effect of average summer rainfall on β.

Some species showed a location (Minnesota, Wisconsin, Michigan) effect, but the limited range of mean stand diameters for many species made its study inconclusive. Some investigations were made on various weighting approaches, but the only one that offered any benefit was to weight proportional to plot size.

Red pine, with 241 measurement sets, had the most data available to study any differences between natural stands and plantations. The plantations tended to have smaller average diameters with the same basic pattern as the natural sets except for a scattering of higher values at about 8 inches. A careful study was done to see if the cause of these higher values could be found, but it was unsuccessful. As a result, no separation as to mode of stand origin (plantation or natural) was made in the final analysis.

In determining a function to fit to the β's, we looked at those species with a fairly complete diameter range. Computing the average β for 2-inch average diameter classes indicated a nearly constant function that decreases somewhat at the lower diameter values. From this relation the function

$$\beta = b_1 (1 - e^{b_2 \overline{DBH}}) \qquad (4)$$

was chosen. A few representative graphs are given in figure 3.

Table 1 contains the coefficients b_1 and b_2 (equation 4) for various species and the number of three-measurement sets on which each is based. Only 490 measurement sets were usable after understocked plots, or those with high mortality or cut, were removed. Measurement sets were also lost because observed crown ratios were used, instead of predicted, whenever possible.

Mixed Stands

The mixed stand analysis proceeded similar to the pure stand analysis, but the model was expanded as follows:

solve

$$\frac{\Delta Y1}{\Delta t} = [PG_1]\,[NT_1]\left[1 - e^{\frac{-\beta_1 = f_1(AD_1)}{(BA_1 + \omega_1 BA_2 AD_2/AD_1)}}\right] \qquad (5)$$

$$\frac{\Delta Y2}{\Delta t} = [PG_2]\,[NT_2]\left[1 - e^{\frac{-\beta_2 = f_2(AD_2)}{(BA_2 + \omega_2 BA_1 AD_1/AD_2)}}\right]$$

with boundary conditions

$Y1(t_1) = k_{11} \quad Y1(t_2) = k_{12} \quad Y1(t_3) = k_{13}$

$Y2(t_1) = k_{21} \quad Y2(t_2) = k_{22} \quad Y2(t_3) = k_{23}$

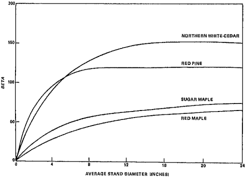

Figure 3. — *Sample monomeolecular functions relating β values to mean stand diameter. Conifer values tend to be considerably larger than hardwood values. The mathematical function is given in equation (4) and the numerical values are given in table 1.*

Table 1.—*Parameter values for equation (4)*[1]

Species	b_1	b_2	Sets of measurement intervals	Comments
Jack pine	112	−.4638	46	Predicted crown ratio used
Red pine	121	−.4462	241	Observed crown ratio used
White pine	117	−.1261	77	Predicted crown ratio used
White spruce	125	−.3652	11	Predicted crown ratio used
Balsam fir	86	−.4923	15	Predicted crown ratio used
Black spruce			9	data only for DBH 2"-4" (use white spruce coefficients)
Tamarack			1	Limited data (use jack pine coefficients)
Northern white-cedar	155	−.2230	18	Observed crown ratio used
Hemlock			2	Limited data (use white pine coefficients)
Black ash			0	Assigned sugar maple coefficients
Cottonwood				
Silver maple	69	−.1434	7	Predicted crown ratio used
Red maple				
Elm			0	Assigned sugar maple coefficients
Yellow birch			1	Limited data (use coefficients for select red oaks)
Basswood			2	Limited data (use coefficients for select red oaks)
Sugar maple	74	−.1902	25	Observed crown ratio used
White ash			0	Assigned sugar maple coefficients
White oaks			4	Limited data (use coefficients for red oaks)
Select red oaks	73	−.1303	10	Observed crown ratio used
Other red oaks			0	Assigned coefficients of select red oaks
Hickory			0	Assigned coefficients of select red oaks
Bigtooth aspen			0	Assigned coefficients of quaking aspen
Quaking aspen	117	−.1366	21	Predicted crown ratio used
Paper birch			0	Assigned coefficients of quaking aspen
Total			490	

[1] For species in which the range of average diameters was small, or only a few observations were available, the estimated beta values were used as a guide in selecting a stubstitute beta function. When no data were available, the species was assigned the beta function parameters for a similar species; for example, we had no pure bigtooth aspen or paper birch plots, so they were assigned the beta function coefficients for quaking aspen.

where

- Y1,Y2 are sums of diameters for species 1 and 2, respectively,
- PG_1,PG_2 are potential diameter growth functions,
- NT_1,NT_2 are numbers of trees,
- AD_1,AD_2 are average diameters of trees in species 1 and 2,
- f_1,f_2 are the monomolecular functions of average diameter (equation 4) for species 1 and 2,
- BA_1,BA_2 are basal area (.005454 $(Y_i/NT_i)^2$) of species 1 and 2, and
- ω_1,ω_2 are the unknown interaction terms that tell how species 2 affects species 1 and how species 1 affects species 2, respectively.

Only the plots judged to be two-species plots (figure 2) were used for this analysis.

We tested the ω values resulting from using both a potential function containing the observed crown ratio and the predicted crown ratio. The results were similar (actually slightly better for edicted crown ratio), as judged by the deviations observations from predictions. Since there were more two-species plots available, we used the preicted crown ratio throughout the two-species analysis.

In the single species analysis we found that plots with less than 40 square feet of basal area had sharply decreased β values. The two-species analysis showed that between 30 and 40 square feet they were nearly the same as above 40 square feet. Thus, we used 30 square feet as the cutoff value for understocked plots instead of 40 square feet to avoid losing additional data. The cutoff criterion for mortality, cut, or ingrowth was 50 percent of initial or surviving trees. Very few plots had to be discarded because of these criteria.

From equation (5) it can be shown that if both ω_1 and ω_2 are exactly 1 and the ratio of average diameters is 1, we can say that species 2 affects species 1 as species 1 affects itself, and species 1 affects species 2 as species 2 affects itself. In this analysis, the following relationships between ω_1 and ω_2 were thought to be most reasonable and were examined:

[1] ω_2 and ω_1 estimated separately
[2] $\omega_2 = 2 - \omega_1$
[3] $\omega_2 = 1/\omega_1$ (6)
[4] $\omega_2 = 2/\omega_1$.

In relationship [1] there is no fixed relation between ω_1 and ω_2, in [2] the sum of ω_1 and ω_2 is equal to 2, in [3] the product of ω_1 and ω_2 is 1, and in [4] the product of ω_1 and ω_2 is 2.

Analysis of mixed plot data for each relationship in equation (6) showed that [1] is undesirable because ω_1 and ω_2 could vary widely with little difference in the goodness of fit of the solution of equation (5) to the observations. (This may have been caused by attempting to estimate four unknowns, ω_1 and ω_2 and initial conditions for Y1 and Y2, based on only six observations of the system state.) Relationship [2] between ω_1 and ω_2 gave very poor results as judged by residual pattern and residual magnitude. Relationships [3] and [4] both gave good results but $\omega_1 \omega_2 = 1$ was slightly better and more logical. Thus, for each two-species mixture we had the constraint that $\omega_1 \omega_2 = 1$. For each plot of, for example, jack pine-red pine, we had an estimated value of ω_1 and ω_2. The mean values for each two-species mixture is given in table 2 along with the number of plots. Because we are dealing with the product of two averages, $\bar{\omega}_1$ and $\bar{\omega}_2$, the product does not necessarily equal 1, but is usually near 1.

Table 2.—*Interaction terms* ω_1 and ω_2 for equation (5)

Species	ω_1	ω_2	Measurement sets available
Jack pine-red pine	2.757	0 483	4
Jack pine-quaking aspen	.829	1 355	3
Red pine-white pine	1.098	1 192	8
Red pine-quaking aspen	.455	2.281	2
White pine-white oaks	.776	1.432	2
White pine-select red oaks	3.500	.286	3
White pine-quaking aspen	1.783	.662	2
Balsam fir-quaking aspen	.581	2.327	16
Black spruce-northern white-cedar	.363	3.066	2
Hemlock-sugar maple	1.598	.977	5
Red maple-yellow birch	859	1.205	2
Red maple-sugar maple	1.571	1.186	4
Red maple-select red oaks	2.321	.589	5
Red maple-paper birch	.389	2 677	2
Elm-basswood	.700	2.010	5
Elm-sugar maple	1.431	.824	11
Elm-white oaks	3.414	293	2
Elm-select red oaks	2.090	.604	7
Elm-quaking aspen	.880	1.441	2
Basswood-sugar maple	.976	1.317	24
Sugar maple-white ash	1.589	.965	9
Sugar maple-select red oaks	2.357	.648	22
White ash-select red oaks	.427	2.346	2
White oaks-select red oaks	.977	1.601	10
White oaks-other red oaks	.282	3.546	2
Select red oaks-other red oaks	1.120	.893	3
Select red oaks-bigtooth aspen	.447	2.497	2
Select red oaks-quaking aspen	.499	2.324	2

DISCUSSION

The unit associated with β (equation 2) is square feet of basal area. Thus, the unit of b_1 in equation (4) is also square feet of basal area. The combination of equations (2) and (4) makes this clear

$$\left[1 - e^{\frac{-b_1 (1-e^{b_2 \overline{DBH}})}{BA}}\right] \qquad (7)$$

From this relation we interpret the β function to give the basal area at which a stand with mean diameter, $\overline{DBH}$, is growing at 63 percent of its potential. That is, when the exponent of e is -1, $1-\frac{1}{e} = 0.63$. On this basis, one would expect tolerant species to have higher function values than intolerant species. The softwoods in table 1 generally follow this pattern although one would expect jack pine to have a lower b_1 value than balsam fir. The hardwood estimates deviate more from the theoretical ordering. One would expect quaking aspen to have the lowest b_1 value. Instead, it has the highest.

Because of the way the model in equation 1 is assembled and the unknown numerical constants estimated, each subsequent step provides a check on previous steps. For example, the β value estimates provide a check on the potential function in the following ways: (1) if the β values are extremely large (recall the β unit is basal area) this means the potential function is predicting growth rates too low, and (2) if the β values exhibit a pattern that contradicts theory, such as quaking aspen having a higher b_1 value than sugar maple, then the potential function for one of the species may need more work. Likewise, when estimating ω_1 and ω_2, a check of the β functions for species 1 and 2, respectively, is made. For example, if two similar species have been observed growing together and the observations are used to estimate ω_1 and ω_2 in the relation

$$\omega_1 \; \omega_2 = 1,$$

one would expect ω_1 to be very close to 1. A value significantly different from 1 would indicate that one of the β functions is not adequately characterizing pure stand growth of the species.

Only pure stands and two-species mixtures were treated because the observational data simply will not support a thorough examination of three-species mixtures. Models for three- and four-species mixtures are built up from two-species models. For example, if we observe species 1 and 2 growing together and estimate ω_{12} and ω_{21} for their interaction, if we observe species 2 and 3 growing together and estimate ω_{23} and ω_{32} for their interaction, and likewise ω_{13} and ω_{31} for 1 and 3 growing together, then a three-species model is formed as follows:

$$\frac{\Delta Y1}{\Delta t} = [PG_1]\,[NT_1]\left[1 - e^{\frac{-\beta_1 = f_1(AD_1)}{(BA_1 + \omega_{12}\, BA_2\, AD_2\, /AD_1 + \omega_{13}\, BA_3\, AD_3\, /AD_1}}\right]$$

$$\frac{\Delta Y2}{\Delta t} = [PG_2]\,[NT_2]\left[1 - e^{\frac{-\beta_2 = f_2(AD_2)}{(BA_2 + \omega_{21}\, BA_1\, AD_1\, /AD_2 + \omega_{23}\, BA_3\, AD_3\, /AD_2}}\right] \qquad (8)$$

$$\frac{\Delta Y3}{\Delta t} = [PG_3]\,[NT_3]\left[1 - e^{\frac{-\beta_3 = f_3(AD_3)}{(BA_3 + \omega_{32}\, BA_2\, AD_2\, /AD_3 + \omega_{31}\, BA_1\, AD_1\, /AD_3}}\right]$$

The explicit assumption here, for example, is that species 2 affects species 3 the same whether or not species 1 is present.

For species mixtures where no information, or insufficient information, is available, a default value of 1 is given to the ω values, pending acquiring additional information.

CONCLUSION

When the potential growth of a plot or stand of trees of a single species is characterized by the product of a potential function, number of trees and a modifier of the potential, the function

$$[1 - e^{-\beta/BA}]$$

satisfactorily reduces the potential growth to that actually observed. The values of β are species specific, being higher for conifers than for hardwoods, and higher for tolerant species than for intolerant species. For stands containing two-species mixtures, variation in the growth was satisfactorily estimated by using two equations:

$$\frac{\Delta Y1}{\Delta t} = [PG_1]\,[NT_1]\left[1 - e^{\frac{-\beta_1 = f_1(AD_1)}{(BA_1 + \omega_1\, BA_2\, AD_2/AD_1)}}\right] \tag{9}$$

$$\frac{\Delta Y2}{\Delta t} = [PG_2]\,[NT_2]\left[1 - e^{\frac{-\beta_2 = f_2(AD_2)}{(BA_2 + \omega_2\, BA_1\, AD_1/AD_2)}}\right]$$

We infer from this that mixtures of three or more species can be handled by expanding the number of equations.

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DIAMETER GROWTH ALLOCATION RULE

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THE PROBLEM

Forest growth models may be categorized in many ways. An often-used dichotomy is stand model vs. tree model. Munro (1974) presented a classification based on the use of inter-tree distances and primary input parameters (variables). In this paper we have added a third dimension to Munro's: projection variables. We feel this is important because one may choose not to project information at the level of detail potentially available, but at an aggregated level. For example, even though a tree list containing d.b.h., species, stem maps, etc. is available as input, one may choose to project the attributes of a class of trees. If, in going from the input form to the projection form, one aggregates tree attributes into class attributes, information is irretrievably lost unless a way can be found to disaggregate change in the projected variables.

There are advantages to maintaining and updating a tree list through time. It allows more useful output because individual trees can be aggregated according to a criterion of interest for output. This is especially important when the output criterion is not the criterion used to aggregate input to the projected variables. For example, output information is often desired according to various merchantability standards, such as 4.5-inch d.b.h. (pulpwood) or 11.0-inch d.b.h. (saw log). There is, however, ample evidence that these size criteria have no biological significance useful for projection purposes.

On the other hand, there are advantages to any approach that aggregates input information into a few classes for projection. Such approaches are more simple and economical, lending greater operational usefulness to the projection system.

The purpose of this paper is to present a simple rule for disaggregating change in a projected variable—i.e., of allocating change by a group of tre to the individual trees in the group. The followin discussion assumes that the basic input information is in the form of a tree list containing tree species and d.b.h., and that this information has been aggregated into a single class for projection purposes.

APPROACH AND HYPOTHESIS

Two fundamentally different ways to approach the allocation (disaggregation) problem may be termed absolute and relative. An example will show the nature of their differences. Assume we have a 12-inch d.b.h. sugar maple on our tree list. It is one of a group of sugar maple trees whose collective attribute (e.g., the sum of their diameters) is projected for 10 years. From values of the collective attributes at the initial and final time we calculate change.

Implementing an absolute allocation rule to estimate the diameter growth on the tree would normally involve either consulting a table of mean diameter growth rates for sugar maple by size or size class, or solving a 10-year diameter growth equation for a 12-inch sugar maple tree. By applying the same procedure to all other trees on the tree list and summing the projected growths, the sum could be compared with the amount of change to be allocated. If, for example, the predicted sum exceeds the amount to be allocated, one could use a reduction constant for all trees.

Implementing a relative allocation rule would, on the other hand, consider the number of trees larger and smaller than the subject tree. According to these numbers, the subject tree is allocated an amount of diameter growth. Because the social environment of a tree is an important determinant of its growth, we have chosen the relative allocation rule.

The hypothesized relative allocation rule tested here is: growth of an individual tree in a stand is to the stand growth as growing stock of the individual tree is to the growing stock in the stand, or $\frac{\Delta d_i}{\Delta d_T} :: \frac{GS_i}{GS_T}$, where the known quantities are

Δd_T, GS_i, GS_T (stand change given by the projection equation, growing stock in the subject tree, growing stock total, respectively), and the unknown quantity is

Δd_i (diameter change of the subject tree).

METHODS

Only calibration data base sample plots judged to be "pure" (Leary and Holdaway 1979) were used for this analysis. The trees on each sample plot were sorted in ascending order by d.b.h. and then grouped into 10 cells of nearly equal frequency, corresponding to the 1-10th percentile, the 11-20th percentile, etc., up to the 91-100th percentile. When the total number of trees, N, was not exactly divisible by 10, the extra trees were added, one per cell starting with the 1-10th percentile cell. Thus, each cell contained approximately 10 percent of the trees on a plot with the smallest 10 percent in the first cell and the largest 10 percent in the last cell.

The contents of each cell were analyzed for two attributes: (1) the growth of the trees in each cell relative to the total growth of all trees on the plot and (2) the growing stock represented by the trees in each cell relative to the total growing stock on the plot. Diameter growth per tree and sum of tree diameters were used as the respective variables.

Proportion of stand growth due the i^{th} cell was computed as follows:

$$\frac{\dfrac{\sum_{j=1}^{k_i} \Delta dbh_{ij}}{k_i}}{\sum_{i=1}^{10} \left[\dfrac{\sum_{j=1}^{k_i} \Delta dbh_{ij}}{k_i}\right]} = y_i \quad (1)$$

where k_i is the number of trees in the ith cell, and Δdbh_{ij} is the periodic d.b.h. growth for the jth tree in the ith cell.

Proportion of growing stock in the ith cell was computed as follows:

$$\frac{\dfrac{\sum_{j=1}^{k_i} dbh_{ij}}{k_i}}{\sum_{i=1}^{10} \left[\dfrac{\sum_{j=1}^{k_i} dbh_{ij}}{k_i}\right]} = x_i \quad (2)$$

where k_i is the number of trees in the ith cell, and dbh_{ij} is the diameter breast height of the jth tree in the ith cell.

These analyses gave two sets of 10 values for each plot having 10 or more trees:

x_i = the proportion of total growing stock in the ith 10-percentile cell, and

y_i = the proportion or total growth due to trees in the ith 10-percentile cell.

Proportion of growth and proportion of growing stock were computed for 10-percentile classes on each plot with 10 or more surviving trees in the time interval selected. Two intervals were selected on each plot: from the final measurement back approximately 10 years, and from the first measurement forward approximately 10 years. If overlap resulted, the plot was not used. A total of 1,599 plot observations were available for analysis. Each of the 1,599 observations consisted of two sets of 10 proportions: growth and growing stock, as well as additional plot descriptors (mean plot basal area at midpoint of the interval, mean stand diameter, site index, property and plot number).

Information on the 1,599 plots was sorted in order by species and mean stand basal area. Preliminary evaluations based on red pine plantation data showed a surprisingly linear relationship between proportion of growth and proportion of growing stock. The same linear relation was evident in the other plantation data available (jack pine and white spruce). Graphs of growth proportion against growing stock proportion for natural stands showed a marked difference from those for plantations. This was especially evident for plots with greater mean d.b.h. values and/or lower stand basal areas. Figures 1a and 1b show the differences in trends as plot basal area changes. Figure 1b is for red pine plantations and figure 1a is for red pine natural stands.

In both figures, there is a tendency for the relation to rotate around or near the point with coordinates .10,.10. This, of course, is to be expected since the plot data were divided into 10-percentile classes. As basal area stocking decreases the figures show three trends: the range in proportion of growing stock decrease, the range in proportion of growth decreases, and the slope of the relation changes from a very steep positive one at 179 square feet in figure 1a to slightly negative at 66 square feet in the same figure. Comparing these trends with those in figure 1b shows that the ranges also decrease, but the slope of the relation is still positive at 50 square feet.

Several approaches were used to develop mathematical equations to characterize the types of changing relationships between proportion of growth and proportion of growing stock. The attempt was basically to estimate parameters for a general equation fit to each set of observations based on basal area classes. For example, a four-parameter model fit to observations for each basal area class would yield one set of parameters (a,b,c,d):

$$GR_{prop} = f(GS_{prop}, a,b,c,d). \qquad (3)$$

Given a set of parameters (a,b,c,d) for each basal area class, the attempt was to develop an expression as follows:

$$a = f(BA), \text{ where}$$

a is a parameter in equation 3,
BA is mean stand basal area in square feet, and
f is an unknown function.

The general function, then, would be

$$GR_{prop}=f(GS_{prop},a=f_1(BA),b=f_2(BA), c=f_3(BA),d=f_4(BA)) \qquad (4)$$

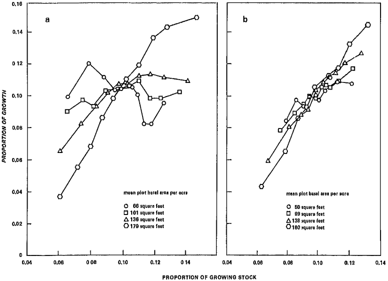

Figure 1.—*Proportion of growth compared with proportion of growing stock for (a) red pine natural stands and (b) plantations in the Lake States.*

where f_i (i = 1,2,3,4) are functions relating basal area (BA) to parameters a,b,c, and d.

This approach resulted in sets of parameter values—for example, a set of six b values that varied so widely that no relation between the b values and basal area could be established. As a result, this approach had to be discarded.

The approach finally used was to take the various graphic forms of relations between growth proportion and growing stock proportion shown in figure 1 and align a series of them on a light table so that they produced, as nearly as possible, a single smooth graph. To this single graph was added a new set of axes. The new coordinates of 17 points were read for segments of the composite graph and used as observations to estimate the parameters in an equation to characterize the composite graph. The function used was:

$$\log(GR_{prop}) = a_0 + b(\log GS_{prop}) + c(\log GS_{prop})^2 + d(\log GS_{prop})^3 \qquad (5)$$

where

log is logarithm to the base 10,

GR_{prop} is growth proportion,

GS_{prop} is growing stock proportion, and

a_0,b,c,d are unknown numerical constants.

Estimated values of the numerical constants are

a_0 = −2.379
b = −5.077
c = −3.972
d = −.2964

(R^2 = 0.986, Standard error = 0.0204). A graph of his function is shown in figure 2.

The strategy in putting together a composite relation is to have a mathematical function that possesses all the required types of shapes. The desired shape for a particular situation can be obtained simply by translating the axes so that the observed data coincide with a segment of the composite function graph. Because of the analogy with he giant tree concept (Grosenbaugh 1954), equaion (6) is called the "giant function".

$$GR=k_1+10^{\,(a_0+b(\log(GS+k_2))+ c(\log(GS+k_2))^2+ d(\log(GS+k_2))^3)} \qquad (6)$$

where a_0,b,c,d have the numerical values given above,

k_1 is the vertical translation of the axes, and

k_2 is the horizontal translation of the axes.

Observations of the growth proportions and growing stock proportions for basal area classes, such as shown in figure 1, were used to determine values of k_1 and k_2 for each basal area class for each species. Table 1 shows k_1 and k_2 values for eight red pine basal area classes for natural stands. The equation used to characterize the relation between k_1 and mean basal area is a simple parabola:

$$k_1=a_1+a_2BA+a_3BA^2 \qquad (7)$$

where

a_1,a_2,a_3 are unknown numerical constants, and the function for k_2 is

$$k_2=b_1(1-e^{b_2(b_3-BA)})b_4 \qquad (8)$$

where

b_1,b_2,b_3 and b_4 are unknown numerical constants.

Constants a_i, i = 1,2,3 and b_i, i = 1,...4 were determined by linear and nonlinear regression methods, respectively, to give the following generalized form to the giant function:

$$GR=k_1+10^{\,(a_0+b(\log(GS+k_2))+ c(\log(GS+k_2))^2+d(\log(GS+k_2))^3)} \qquad (9)$$

where a_0,b,c,d are known constants for the giant function, and k_1 and k_2 are computed from plot basal area using equations (7) and (8).

In most cases it was necessary to examine the observations and assign numerical values to b_1 and b_3 in equation (8). Table 2 gives the coefficients of equations (7) and (8) for all species with sufficient data.

Table 1.—*Vertical (k_1) and horizontal (k_2) translation values for giant function to account for variation in relation between growth proportion and growing stock proportion for red pine natural stands (groups are by basal area classes)*

Basal area		k_1	k_2	R^2	No. of plots
Mean	Range				
65.6	(20-69)	-.1060	.1346	.42	8
79.5	(70-88)	-.1111	.1174	.44	21
101.5	(90-109)	-.1099	.0961	.53	52
119.1	(110-129)	-.1043	.0826	.93	151
137.9	(130-149)	-.1009	.0734	.97	86
156.0	(150-166)	-.0985	.0673	.96	39
178.9	(170-189)	-.0565	.0315	.99	19
197.6	(191-207)	-.0689	.0396	.71	5

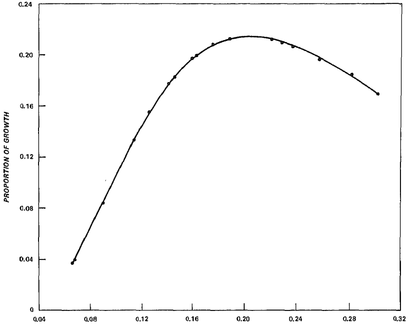

Figure 2.—*Graph of composite function in equation 5 showing location of 17 observations from separate graphs.*

RESULTS AND DISCUSSION

The functions to predict translation values, k_1 and k_2, show a considerable difference in the pattern of translations as basal area increases. Figure 3 shows the patterns for Lake States species with sufficient plots judged to be pure. Species differences relate primarily to the magnitude of k_1 and k_2, and the rate at which k_1 and k_2 approach zero. In general, tolerant species' values approach zero very slowly while intolerants rapidly approach zero. Quaking aspen appears to be an exception to this statement.

Small differences between species' k_1 and k_2 values can make a sizable difference in the relation between growth proportion and growing stock proportion. For example, the k_1 and k_2 values for red pine natural stands and plantations (figs. 3g and 3h, respectively) are very similar, yet the pattern for 3g produces the relations in figure 1a and that for 3h produces the relations in figure 1b.

The total sequence of operations in predicting growth proportion is given schematically in figure 4. Four input quantities are required for a given tree and stand to determine the tree's d.b.h. growth: total stand growth (from the stand component projection equation), d.b.h. of the *i*th tree, sum of tree diameters on the plot, and the total stand basal area.

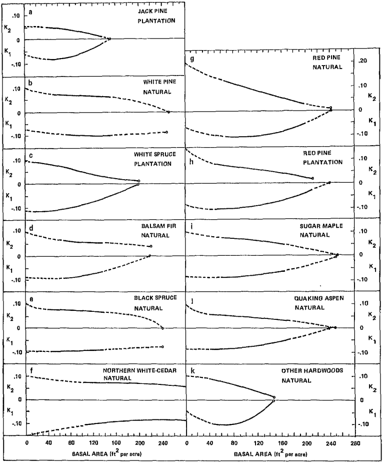

Figure 3.—*Translation values k_1 and k_2 for Lake States species. The solid lines denote the range of basal area for which data were available. The dotted lines are extrapolations based on equations (7) and (8) using coefficients in table 2.*

Table 2.—*Coefficients of equations (7) and (8) used to predict vertical and horizontal translations of axes so that appropriate section of giant function is used to characterize growth proportion–growing stock proportion*

	k_1 (vertical)			k_2 (horizontal)			
Species	a_1	a_2	a_3	b_1	b_2	b_3	b_4
Jack pine (plantation)	-.630 E-01	-.730 E-03	.770 E-05	.10 E 00	-.4000 E-02	.150 E+03	.7400 E 00
Red pine (plantation)	-.844 E-01	-.540 E-03	.380 E-05	.15 E 00	-.5585 E-02	.300 E+03	.2139 E+01
(natural)	-.643 E-01	-.968 E-03	.500 E-05	.20 E 00	-.9400 E-02	.300 E+03	.4102 E+01
White pine (natural)	-.736 E-01	-.350 E-03	.140 E-05	.10 E 00	-.7300 E-03	.250 E+03	.1759 E 00
White spruce (plantation)	-.111 E 00	-.100 E-03	.339 E-05	.10 E 00	-.1799 E-01	.250 E+03	.8264 E+01
Balsam fir (natural)	-.871 E-01	-.242 E-03	.283 E-05	.10 E 00	-.1720 E-02	.250 E+03	.3479 E 00
Black spruce (natural)	-.100 E 00	.620 E-04	.800 E-07	.10 E 00	-.3190 E-02	.250 E+03	.3724 E 00
No. white-cedar (natural)	-.154 E 00	.583 E-03	-.120 E-05	.10 E 00	-.2018 E-02	.400 E+03	.3724 E 00
Sugar maple (natural)	-.840 E-01	-.240 E-03	.240 E-05	.10 E 00	-.6200 E-02	.250 E+03	.8322 E 00
Quaking aspen (natural)	-.819 E-01	-.247 E-03	.248 E-05	.10 E 00	-.3470 E-02	.250 E+03	.7540 E 00
Other hardwoods (natural)[1]	-.394 E-01	-.196 E-02	.150 E-04	.10 E 00	-.2297 E-01	.200 E+03	.6384 E+01

[1] Red maple, elm, basswood, yellow birch, white oak, select red oak

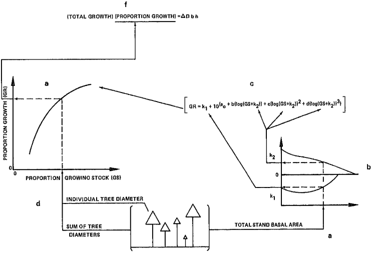
f
(TOTAL GROWTH) (PROPORTION GROWTH) =ΔD b h
a
PROPORTION GROWTH (GR)
0
0
PROPORTION GROWING STOCK (GS)
c
GR = k1 + 10(ao + b(log(GS+k2)) + c(log(GS+k2))2 + d(log(GS+k2))3)
k2
b
0
k1
d
INDIVIDUAL TREE DIAMETER
SUM OF TREE DIAMETERS
TOTAL STAND BASAL AREA
a

Figure 4.—*Sequence of steps in the estimation of diameter growth for an individual tree in a stand. Steps a, b, and c establish the relation between growth proportion and growing stock proportion. Steps d, e, and f apply the relation to all trees in a stand. Total growth at step f comes from the projection equation.*

In the projection procedure each tree is treated individually rather than being grouped into 10-percentile classes. Considering a single tree on a plot of N trees is equivalent to dividing the plot into $\frac{1}{N}$ 100-percentile classes. In this case the relation between proportion of growth and proportion of growing stock should have the same shape as the 10-percentile graph but should be centered at $\frac{1}{N}, \frac{1}{N}$ instead of (.10,.10). This can be accomplished by transforming the per-tree growing stock value to a 10-percentile class value. The latter value can be used as in figure 4, step e, and then be transformed from a 10-percentile growth proportion value to a $\left(\left(\frac{1}{N}\right)100\right)$ percentile growth proportion value. The conversion relations are based on the following proportions:

$$\frac{x_0}{X} :: \frac{\frac{1}{N}}{.10} \text{ or } X = x_0\,(.10)N$$

where

x_0 is the per tree growing stock proportion,
N is the number of trees, and
X is the 10-percentile growing stock proportion.

The value X is used as in figure 4e and a 10-percentile growth proportion is computed. This value is transformed back to a $\left(\left(\frac{1}{N}\right)100\right)$ percentile basis using the following relation:

$$\frac{y_0}{Y} = \frac{\frac{1}{N}}{.10}\text{ , or } y_0 = \frac{Y}{(.10)N}$$

where

y_0 is the per tree growth proportion,
N is the number of trees, and
Y is the 10-percentile growth proportion.

Because of the rather complicated procedures used to estimate all the giant function parameters, we used observed growing stock proportion and observed basal area, and computed a predicted growth proportion. These values were compared with the observed growth proportion. It appeared that no systematic bias had been introduced into our procedures because the deviations of predicted from observed were as a rule normally distributed about zero for both the percentile class and stand basal area. This test does not, of course, constitute an independent test; rather, it simply checks for bias introduced by our procedures.

CONCLUSION

The growth of an individual tree in a stand is a function of its position in the stand and of the stand growth. The initial hypothesis

$$\frac{\text{tree growth}}{\text{stand growth}} :: \frac{\text{tree growing stock}}{\text{stand growing stock}}$$

is overly simplistic. The growth proportion is related to growing stock proportion in a nonlinear manner that changes with basal area stocking levels. Furthermore, the nonlinear relation differs between natural and plantation stands of the same species.

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MORTALITY FUNCTIONS

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Knowing an individual tree's likelihood of dying within the next year is essential to a tree-model forest growth simulator. A mechanism for selecting trees to be removed from a stand as a consequence of mortality is an integral feature of the North Central Forest Experiment Station's generalized forest growth projection system (Leary 1979).

A tree's probability of dying depends upon its ability to cope with its environment. The tree itself, its neighboring trees, and numerous additional influences including light, moisture, nutrients, insects, and fungi combine to form the unique niche for the individual tree.

How might we assess the tree's response to the conditions of its neighborhood? One of the simplest indicators is the tree's vigor as reflected in its growth. Diameter growth rate is a readily obtainable, direct measure of the tree's ability to withstand the stresses of life.

Diameter growth rate is the key tree characteristic in the species-specific mortality functions presented in this paper. We attempted to select a common functional form for all species, with unique parameters for each species.

DATA ORIGIN

Permanent growth plot data were available for the major forest tree species in Minnesota, Wisconsin, and Michigan (Christensen *et al.* 1979). Most individual tree records contained measurements for at least three points in time, permitting calculation of an annual mortality rate for a given growth rate. Determining diameter growth rate required two diameter measurements; determining the tree's status (live or dead) and subsequently, the mortality rate, required a third measurement. The time between measurements ranged from 1 year to more than 10, with the most frequent interval being 5 years.

Intervals of at least 2 years but no more than 6 were chosen to establish a reliable measure of diameter growth rate. A comparable time interval (2 to 6 years) following growth rate determination was chosen for recording tree status.

These criteria reduced the size of the data base eligible for use in developing the mortality function. Even so, more than 47,000 trees (table 1) had the required three measurements. The tree count ranged from 18,208 for red pine and 7,350 for sugar maple down to about 100 for white spruce, tamarack, and bigtooth aspen.

Furthermore, many of the trees had more than three measurements that met the time requirements. However, only one set of measurement data—one diameter growth rate followed by a status observation—was used from each tree when deriving the mortality function. This set was chosen at random for each tree where two or more sets were available.

The number of trees by species in the final mortality data base is presented for each diameter growth rate class (table 1) and for each diameter class (table 2).

MORTALITY RATE

Individual trees either lived, (mortality zero), or died, (mortality 1.0). Diameter growth class mortality rates, however, depended upon the composite of the individual rates of those trees forming the class.

Growth class limits chosen were 0.0 inch (no growth), 0.02 inch, 0.04 inch, and so on up to 0.30

Table 1.—*Number of trees used in estimating mortality rates and parameters, by 5-year diameter grow classes*

	Five-year diameter growth class (inches)								
	.00-.09	.10-.19	.20-.29	.30-.49	.50-.69	.70-.89	.90-1.0	Over 1.0	Tot
Jack pine	104	165	178	492	833	763	718	528	3,7
Red pine	528	605	1,033	3,987	4,571	3,663	2,040	1,781	18,2
White pine	49	44	55	130	139	189	170	407	1,1
White spruce	3	2	6	10	12	25	9	39	1
Balsam fir	118	207	234	562	648	431	271	331	2,8
Black spruce	12	34	59	123	96	43	21	11	39
Tamarack	1	5	6	16	19	8	3	3	6
N. white-cedar	0	4	11	42	53	30	5	5	15
Hemlock	17	23	34	92	68	61	39	22	35
Black ash	8	10	21	45	42	23	10	5	16
Red maple	36	35	52	193	246	221	166	185	1,13
Elm	168	65	141	258	241	204	234	374	1,68
Yellow birch	38	32	51	143	129	100	51	53	59
Basswood	103	59	110	227	262	286	328	348	1,72
Sugar maple	402	216	538	1,145	1,447	1,347	1,214	1,041	7,35
White ash	20	22	60	129	131	182	237	319	1,10
White oak	33	29	50	117	103	66	30	18	44
Select red oaks	19	37	50	216	323	263	250	235	1,39
Other red oaks	11	4	21	28	32	20	13	11	14
Hickory	22	24	60	114	67	29	13	7	33
Bigtooth aspen	0	1	4	20	20	27	20	16	10
Quaking aspen	105	213	198	398	382	389	443	791	2,91
Paper birch	64	86	91	165	167	99	78	55	80
Other hardwoods	16	20	48	54	29	36	20	26	24
Noncommercial	101	22	70	77	24	10	3	2	30
Total									47,50

inch per year. These limits correspond to 5-year growth remeasurements (as mentioned earlier, 5-year remeasurements prevailed in the data base) of zero diameter growth, 0.1 inch, 0.2 inch, etc.

The mortality rate (R), the annual mortality for a growth class, was calculated from the more readily determined annual survival rate (SR), with R=1−SR.

If we assume the SR for a species to be the same each year of the 2- to 6-year status measurement interval, while allowing a tree to die but once with its demise being independent of others of the species, we have

$$\sum S_i = \sum N_i SR^i . \qquad (1)$$

In (1) i is the length of the status observation interval, S_i is the number of trees surviving the i^{th} interval, and N_i is the number of live trees at th beginning of the i^{th} interval.

These terms become more meaningful if w examine real status data, for example, the statu of 528 "no growth" red pine trees:

Number of live trees (N_i) at second measurement	Number of years (i) to third measurement (status)	Number of live trees (S_i) at status measurement
0	2	0
16	3	13
31	4	29
417	5	347
64	6	44

Table 2.— *Number of trees used in estimating mortality rates and parameters, by d.b.h. class*

Species	D.b.h. in inches at second measurement (prior to status observation)															
	to 1.5	1.6-2.5	2.6-3.5	3.6-4.5	4.6-5.5	5.6-6.5	6.6-7.5	7.6-8.5	8.6-9.5	9.6-10.5	10.6-11.5	11.6-12.5	12.6-13.5	13.6-14.5	14.6-15.5	15.6-16.5	16.6& larger	Total
Jack pine	62	426	851	482	477	509	362	199	159	103	51	42	20	16	14	4	4	3,781
Red pine	12	91	234	928	1,937	2,889	2,851	2,221	1,811	1,491	1,124	853	628	432	320	183	203	18,208
White pine				23	72	64	70	70	76	75	81	81	78	82	79	71	261	1,183
White spruce	2	11	12	10	7	9	19	12	10	5	3	1	1	2	1		1	106
Balsam fir	92	596	631	463	298	303	207	107	61	22	12	5	4	1				2,802
Black spruce	3	12	35	44	68	132	80	16	6	3								399
Tamarack		3		1	9	20	11	10	3	2	2							81
N. white-cedar			1	1	3	36	34	21	27	15	6	4		1			1	150
Hemlock					30	36	44	33	35	30	33	24	23	26	16	12	14	356
Black ash				9	17	47	33	20	7	14	8	5	1	1	2			164
Red maple		7	3	19	97	237	185	156	115	92	74	50	22	28	15	10	24	1,134
Elm		18	69	119	156	178	180	175	172	155	136	85	65	46	35	33	63	1,685
Yellow birch		1		6	62	140	114	61	56	54	28	12	22	15	7	8	11	597
Basswood		20	33	68	148	225	220	195	191	170	134	72	54	33	32	27	101	1,723
Sugar maple	23	252	269	743	1,123	1,324	1,028	705	486	336	234	187	135	113	101	86	205	7,350
White ash		10	20	51	86	150	139	154	142	121	94	61	29	16	11	5	11	1,100
White oak		2			10	5	28	39	49	60	45	49	26	29	29	17	58	446
S red oak				1	26	72	87	90	85	109	134	120	118	119	79	83	270	1,393
Other red oaks	3	7	10	13	12	6	6	3	6	12	17	9	6	9	6	3	12	140
Hickory		2	4	7	11	14	50	82	70	34	29	17	6	5	3		2	336
Bigtooth aspen					3	20	18	15	5	12	19	11	3	1				108
Quaking aspen		21	98	313	304	439	513	448	300	234	147	57	29	7	4	4	1	2,919
Paper birch		169	128	72	77	132	113	55	29	14	6	4	2	1	2		1	805
Other hardwoods		10	3	11	25	30	38	25	26	18	23	18	6	7	3		6	249
Noncommercial		28	39	51	85	49	30	15	7	5								309

If the status observation follows the second diameter measurement by 2 or more years and the status interval is of common length for all trees (i.e., one level of i), then (1) reduces to $S = N\,SR^{YR}$, where YR corresponds to the length, in years, of the observation interval. This yields $SR = (S/N)^{1/YR}$ and can be solved by substituting the observed N, S, and YR in the function. Based on the second line in the tabulation (i = 3), the annual survival rate, SR, is $(13/16)^{1/3}$ or 0.933. Calculating further, SR = 0.983 for i = 4, 0.964 for i = 5, and 0.939 for i = 6.

When several different status interval lengths are encountered, as with i = 3, 4, 5, and 6, application of equation (1) will yield a composite SR for all intervals. Substituting observed values in $\sum S_i = \sum N_i SR^i$ yields $433 = 16\,SR^3 + 31\,SR^4 + 417\,SR^5 + 64\,SR^6$. Solving, we find SR to be 0.9611 and R to be 0.0389.

We then have $\sum S_i = 433$ and $\sum N_i SR^i = 433.05$ with the red pine data, both a check on our calculations and the completion of the demonstration. In summary, we have:

N_i	i	S_i	$N_i SR^i$
0	2	0	0.00
16	3	13	14.20
31	4	29	26.45
417	5	347	341.96
64	6	44	50.44

Inspection of each species' mortality data revealed that decreasing mortality was associated with increasing diameter growth. When diameter growth exceeded 0.5 inch in 5 years, the mortality rate essentially leveled off without reaching zero. Tamarack and northern white-cedar were exceptions; these species had consistently low mortality at all growth levels.

There are four critical points where reliable mortality data are needed. Foremost is the mortality rate for "no growth" trees. If the data at this point need to be bolstered by including trees showing slight growth, then the estimate of mortality at no growth loses some reliability.

A second critical data point is the mortality rate for the fast-growing trees. For each species the most ample data appeared here.

At least two reliable data points between the nongrowers and the fast-growers are needed to establish the form of the mortality curve as mortality declines.

Mortality rates for at least the four critical diameter growth rates were established for all species except other red oaks, tamarack and northern white-cedar. With tamarack and northern white-cedar the relationship of mortality to diameter growth was not established because too few trees were available, especially too few slow or no growth trees. Therefore, a uniform mortality rate was assigned to all tamarack and another to northern white-cedar.

MODEL SELECTION

A mathematical expression for relating mortality to growth rate was sought for each species. The logistic function $Y = [1+\mathrm{EXP}(\alpha+\sum\beta X)]^{-1}$ has been used effectively for modeling tree mortality (for example, Hamilton and Edwards (1976) and Monserud (1976) and for modeling human health risk (for example, Truett *et al.* (1967)).

Writing the logistic in terms of our application we have

$$R = [1+\mathrm{EXP}(\alpha+\beta{\bullet}\mathrm{DGR})]^{-1}, \tag{2}$$

with DGR being the annual diameter growth rate.

Equivalent forms are $R' = \alpha'\ \mathrm{EXP}\ (\beta{\bullet}\mathrm{DGR})$ and $R' = \mathrm{EXP}\ (\alpha+\beta{\bullet}\mathrm{DGR})$, with $R' = (1 - R)/R$.

Three variations from this basic function were also evaluated for usefulness in estimating mortality:

$$R = [1+\mathrm{EXP}\ (\alpha+\beta{\bullet}\mathrm{DGR})]^{-1} + \delta, \tag{3}$$
$$R = [1+\mathrm{EXP}\ (\alpha+\beta{\bullet}\mathrm{DGR}^{\gamma})]^{-1},\ \text{and} \tag{4}$$
$$R = [1+\mathrm{EXP}\ (\alpha+\beta{\bullet}\mathrm{DGR}^{\gamma})]^{-1} + \delta. \tag{5}$$

Still another model was considered,

$$R = [1+\mathrm{EXP}\ (\alpha+\beta{\bullet}\mathrm{DGR}^{\gamma}+\omega{\bullet}\mathrm{DBH})]^{-1} + \delta, \tag{6}$$

to accommodate those species in which the effect of DGR depended upon tree size.

In these models, the parameter, α, establishes mortality at the no-diameter growth level with the very minor contribution of δ. DBH and its multiplier, ω, provide an adjustment based on tree size especially for trees having little or no growth. Inclusion of a background death rate, δ, provides for the occasional removal of a fast-growing tree. The progression of the mortality estimates from no-growth trees to that of fast growers is controlled by β and γ. Adding the exponent, γ, greatly increases the function's flexibility.

Model (6) was selected for seven of the species (table 3) and model (5) for the remainder. Estimates of the parameters under (5) are given for all species (table 4) to facilitate parameter comparison, even though (6) is the recommended model for seven species.

A preliminary screening of models (2) through (5), the first step in the selection process, was based on the ability of each to reproduce the mortality observed for each diameter-growth class. Inclusion of the background death rate (δ), models (3) and (5), greatly improved the estimates for annual growth rates in excess of 0.1 inch where a low, but non-zero, mortality rate was observed.

Table 3.— *Mortality function[1] for selected Lake States tree species (estimates based on diameter and diameter growth rate)*

	Parameters				
Species	α	β	γ	ω	δ
Jack pine	0.3678	60.178	1.0727	0.12220	0.00502
Red pine	1.0843	11.012	.5887	.48500	.00020
Balsam fir	2.6976	30.550	.8996	-.23069	.00400
Basswood	-.5861	101.870	1.6493	.68398	.00489
Sugar maple	.8150	66.937	1.5934	.75000	.00435
Other red oaks	-1.1492	9.109	2.4552	.35000	.00000
Quaking aspen	-.0786	213.630	1.7126	.25812	.02276

[1] Estimated annual probability of a tree dying is $\{1/[1 + \mathrm{EXP}\ (\alpha + \beta \cdot \mathrm{DGR}^{\gamma} + \omega \cdot \mathrm{DBH})]\} + \delta$ with DGR = diameter growth rate (inches per year) and DBH = diameter breast high.

The variable exponent for diameter growth rate (γ) yielded exponents less than 0.75 for three species, and more than 1.25 for 10 species (table 4)—a substantial departure from 1. In these instances model (5) more realistically portrayed the decline in mortality for trees with no growth to trees with 0.1-inch annual growth than did model (3).

Finally, model (5) could well be superior to the others eliminated at this point, yet still be deficient. Numerous questions come to mind. Is it applicable for all sites? Is it applicable over the geographic range of the species, for all stand densities, and for all diameters? And more.

One major evaluation was completed; estimated tree mortality was compared to mortality observed within each diameter class. To accomplish this, a two-way table was constructed having 1.0-inch diameter classes as one dimension and 0.02-inch annual growth classes as the other. Observed and estimated tree deaths along with total trees were calculated for each cell as well as for each diameter class and each diameter growth class.

Disagreement between observed and estimated tree deaths could be accepted, and was expected, within a diameter class. However, if these discrepancies formed a pattern whereby the estimated deaths were consistently too low for small trees and too high for large trees, or if the reverse pattern were true, then model (5) was deemed inadequate.

For six species, including red pine, balsam fir, basswood, sugar maple, other red oaks, and quaking aspen, model (5) overestimated mortality on the small trees and underestimated on the large. Jack pine revealed a modest reversal from this pattern. Model (6) adequately adjusted the estimated diameter class mortality patterns. Hence we used model (6) for these species.

Table 4.—*Mortality functions*[1] *for Lake States tree species (estimates based on diameter growth rate)*

	Parameters			
Species	α	β	γ	δ
Jack pine	1.2178	149.3669	1.3699	0.00856
Red pine	3.2185	17 3086	.6165	.00042
White pine	2.3507	33.6094	1.0088	.00201
White spruce	2.8806	30.7737	1.2309	.00396
Balsam fir	2.0754	29.5772	.9541	.00265
Black spruce	2.8774	432.9522	1.8576	.00235
Tamarack	13.0000[2]	1.0000	1.0000	.01334
N. white-cedar	13.0000[2]	1.0000	1.0000	.00402
Hemlock	2.8233	89 5603	1.0966	.00320
Black ash	1.4875	13.3384	.5429	.00018
Red maple	2.2405	53.5938	1.0392	.00494
Elm	1.9889	44.5414	1.0965	00899
Yellow birch	2.6705	63.1477	1.3819	.00382
Basswood	2.2779	25.3602	.9027	.00317
Sugar maple	3.1831	82.5642	1.3605	.00405
White ash	1.7821	232.3075	1.7267	.00222
White oak	3.6946	95.6369	1.4517	.00188
S. red oak	2.3092	6,999.8220	2.6615	.00468
Other red oaks	1.0044	69.2301	1.8048	.00000
Hickory	3.0626	63.4946	1.0498	.00134
Bigtooth aspen	.8805	56.5925	1.2154	.01288
Quaking aspen	1.2199	693.5903	2.0753	.02384
Paper birch	2.1808	11.6263	.4750	.0002
Other hardwoods	2.2339	29.6290	1.3427	0074
Noncommercial	2.3581	113.0644	.9993	.0192

[1] Estimated annual probability of a tree dying is $\{1/[1+\mathrm{EXP}(\alpha+\beta \cdot \mathrm{DGR}^{\gamma})]\} + \delta$ with DGR = diameter growth rate (inches per year).

[2] Mortality is constant for all growth rates, relation to DGR not established, no slow-growing trees in sample.

VALIDATION

To evaluate model performance the mortality models were applied to two data sets that were not utilized in model development. One set consisted of 266 permanent plots from the Cloquet Experimental Forest, Carlton County, Minnesota; the Forest is managed by the College of Forestry, University of Minnesota. These plots provided test data primarily for aspen, birch, and northern conifers (table 5). An initial measurement and three remeasurements, spanning a period of 17 years, were available.

The second set consisted of 38 permanent plots from the Dukes Experimental Forest, Marquette County, Upper Michigan. The Dukes Forest is managed by the North Central Forest Experiment Station. These plots provided a small set of test data for northern hardwoods (table 5). An initial measurement and two remeasurements, spanning a period of 10 years, were available.

One validation consisted of comparing the number of trees, by species, expected to die during the status observation interval with those actually dying in the interval. With this, the probability of death was determined for each tree of the species using that species' mortality model. As in the model derivation process, diameter growth rate was based on the first two diameter measurements for a tree, and the status was observed at the third measurement. The count of trees observed to die was compared to expected deaths based on the model (table 5).

The largest discrepancy between actual tree deaths and model estimates was for jack pine. Excessive deaths were yielded by the model, especially for trees growing less than 0.08 inch per year, and for trees less than 8.6 inches in diameter. The tamarack and quaking aspen models also overestimated mortality, though the aspen differences were not large.

Table 5.— *Observed and estimated tree mortality on validation plots by tree species*

	CLOQUET PLOTS					
	Initial measurement through second remeasurement[1]			First remeasurement through third remeasurement[2]		
Species	Trees in test	Mortality		Trees in test	Mortality	
		Observed	Estimated		Observed	Estimated
Jack pine	1,674	111	211	1,597	207	278
Red pine	899	3	5	1,032	6	17
White pine	170	11	6	185	20	9
White spruce	69	6	4	104	0	6
Balsam fir	513	73	65	613	130	94
Black spruce	778	54	55	832	118	81
Tamarack	300	6	19	318	16	29
N. white-cedar	354	7	7	394	16	11
Ash	61	1	5	76	13	8
Red maple	75	0	3	93	8	5
Bigtooth aspen	53	8	10	50	10	14
Quaking aspen	611	89	103	653	166	174
Paper birch	800	8	12	889	39	24
	DUKES PLOTS[3]					
Elm	71	1	4	—	—	—
Basswood	67	1	2	—	—	—
Sugar maple	400	5	6	—	—	—

[1] Diameter growth rate based on 5 years' growth, status observed 5 years later.

[2] Diameter growth rate based on 5 years' growth, statue observed 7 years later.

[3] Diameter growth rate based on 7 years' growth, status observed 3 years later.

Numerous species, including white spruce, northern white-cedar, bigtooth aspen, elm, basswood, and sugar maple, showed good agreement between actual and estimated mortality. With the exception of 10 predicted deaths among the 148 1-inch trees (none were found), the red pine estimates were good. Ash and red maple showed variation in results between observation periods, but on balance yielded good results.

Mortality was underestimated for white pine, primarily with the 6-inch trees; for balsam fir and black spruce, primarily with the 5-, 6-, and 7-inch trees; and for paper birch.

The second validation consisted of simulating mortality for each plot, paralleling the use of mortality models in stand growth simulation. Based on random selection in proportion to each tree's likelihood of death, each tree was assigned either a mortality or a survivor status. A count of these mortality trees provided an estimate of plot mortality through simulation for comparison with observed plot mortality.

To arrive at the probability associated with each mortality count, each tree on a plot was given 100 assignments of 1 (mortality tree) or 0 (survivor tree). This was done through generating 100 random numbers between 0 and 1 for each tree and comparing each number with the tree's likelihood of death. If the random number did not exceed this likelihood, the tree was assigned a status of 1, death. Otherwise the tree was scored 0, a survivor tree.

The first scores were added for all trees on the plot to arrive at one mortality count for the plot. This process was repeated for each of the 99 remaining scores. A distribution of these 100 counts was made for each plot, providing a relative frequency or probability for each mortality count.

Consider the results from Cloquet plot 82 (table 6). This plot had 30 trees with two diameter measurements. When status was recorded 5 years later, one dead tree was found. Expected mortality using the mortality functions for the species present was 0.94. On this plot the probability of selecting exactly one tree to die in the 5-year interval based on 100 applications of the selection process was 0.30, i.e., 30 of the 100 simulation runs produced exactly one dead tree. The probability of selecting either zero, one, or two trees, a count within one of the observed mortality, was 0.91.

Ten Cloquet plots demonstrate the discrepancies that might be encountered in stand growth simulation (table 6). These plots were selected because of their close agreement between observed and expected mortality. Still, the highest proportion of hits, simulation results equal to observed mortality, was the 0.47 (47 of the 100 simulation

Table 6.—*Relative frequencies (probabilities) of mortality tree counts for 10 Cloquet plots determined by random selections in proportion to each tree's probability of dying in 5 years (basis: 100 sets of tree counts per plot)*

		Mortality trees		Mortality tree counts													
Plot	Trees	Observ.	Exp.[1]	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Number																	
82	30	1	0.94	.43	.30	.18	.08	.01	—	—	—	—	—	—	—	—	—
68	19	1	1.01	.36	.42	.17	.05	—	—	—	—	—	—	—	—	—	—
237	30	1	1.00	.34	.47	.16	.03	—	—	—	—	—	—	—	—	—	—
4	36	2	1.55	.26	.28	.27	.15	.02	.01	.01	—	—	—	—	—	—	—
398	18	2	2.43	.01	.18	.40	.18	.16	.05	.02	—	—	—	—	—	—	—
147	53	3	3.48	.02	.10	.12	.25	.17	.15	.10	.07	.02	—	—	—	—	—
111	33	4	4.63	.01	.01	.11	.13	.21	.24	.17	.08	.02	.02	—	—	—	—
285	29	5	4.65	—	.02	.11	.17	.23	.21	.13	.06	.05	.02	—	—	—	—
177	36	5	4.92	—	.01	.05	.12	.21	.23	.21	.14	.02	.01	—	—	—	—
204	45	7	7.88	—	—	.01	—	.02	.13	.18	.17	.16	.17	.10	.03	.02	.01

[1] Expected mortality is the sum of the tree probabilities of dying over 5 years.

runs produced one dead tree each) found for plot 237. Plot 177, with observed mortality at five trees and expected at 4.92, showed only 23 hits in 100 applications.

Clearly, the mortality counts from stand growth simulation will often differ from the actual, even when the model is appropriate. However, the demonstration material (table 6) indicates we can expect to be within one tree of the true count most of the time.

The Cloquet and Dukes plots having at most one mortality tree were summarized on the basis of conformity between each plot's observed mortality and the results from mortality simulation (table 7). On four plots (Cloquet plots having 11 or more trees, one with no mortality and three with one mortality tree each) the probability of getting the correct mortality count through simulation was zero. At the other extreme, 26 of the Cloquet plots having 10 or fewer trees and no mortality showed probability of 0.91 to 1.0 of getting the correct mortality through simulation.

Except for plots having 10 or fewer trees and no mortality, most plots did not have a high probability of hitting the observed mortality through mortality simulation. However, most plots met the relaxed criterion of being within one tree of the correct number.

Only 31 of the 87 Cloquet plots with two or more mortality trees had a probability exceeding 0.20 of producing the correct mortality count through simulation (table 8). Furthermore, the criterion of being within one tree of the correct count was not met consistently by plots having three or more mortality trees.

Thirteen plots containing 11 or more trees each had zero probability of producing the correct mortality count through simulation. Of these, 11 had mortality counts in excess of the observed mortality, and two were less than the observed mortality. Jack pine was the prime contributor to nine of the plots having excessive counts.

Table 7.— *Probability of conformity by plot between mortality tree counts determined by random selections in proportion to each tree's probability of dying (simulation) and observed counts for each plot having at most one mortality tree (basis: 100 sets of simulation tree counts per plot)*

	(Number of plots)											
	Cloquet plots								Dukes plots[2]			
	Plots: 10 or fewer trees				Plots: 11 or more trees							
Probability	Mortality (0)[1]		Mortality (1)		Mortality (0)		Mortality (1)		Mortality (0)		Mortality (1)	
	Hit[3]	Within 1 tree	Hit	Within 1 tree	Hit	Within 1 tree	Hit	Within 1 tree	Hit	Within 1 tree	Hit	Within 1 tree
.00					1		3	2				
.01-.10			3		13	2	5	2	1		1	
.11-.20			2		8		5	1				
.21-.30			2		6	5	7	2	1		2	
.31-.40			1		5	5	11		2		2	
.41-.50	3		2		5	5	5	4	2	1		
.51-.60	1				5	4		2	4			
.61-.70	2				6	5		3	7	1		
.71-.80	6	1			10	5		2	5	4		
.81-.90	16	3	1		4	11		5	8	3		
.91-1.0	26	50		11	8	29		13	2	23		5
Total plots	54	54	11	11	71	71	36	36	32	32	5	5

[1] No mortality observed on these plots.

[2] One plot with two mortality trees not included.

[3] The simulation count for the plot is the same as the observed mortality.

Table 8.— *Probability of conformity by plot between mortality tree counts determined by random selections in proportion to each tree's probability of dying (simulation) and observed counts for each Cloquet plot having two or more mortality trees (basis: 100 sets of simulation tree counts per plot)*

	(Number of plots)											
	Plots: 10 or fewer trees			Plots: 11 or more trees								
	Mortality (2 or 3)[1]			Mortality (2)			Mortality (3)			Mortality (4 or more)		
Probability	Hit[2]	Within 1 tree	Within 2 trees	Hit	Within 1 tree	Within 2 trees	Hit	Within 1 tree	Within 2 trees	Hit	Within 1 tree	Within 2 trees
.00	2			2			4	2	1	3	1	1
.01-.10	5	1		3	2	1	6	3	2	15	5	
.11-.20	1	1		9	1	1	3	1		3	5	3
.21-.30	3	3		10	1		5	2	1	7	5	4
.31-.40		2		5	2	1	1	3	1		4	1
.41-.50					1			2	2		1	4
.51-.60		1			3	1		2	1		5	3
.61-.70					12	1		3	2		2	5
.71-.80		2			5			1	3			1
.81-.90					2	5			4			4
.91-1.0		1	11			19			2			2
Total plots	11	11	11	29	29	29	19	19	19	28	28	28

[1] Either two or three mortality trees observed on these plots.

[2] The simulation count for the plot is the same as the observed mortality

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TIMBER MANAGEMENT GUIDES AND MARKING RULES[1]

Gary J. Brand, *Research Forester*

Previous papers have described a forest growth projection system capable of "growing" and "killing" trees. Because it does not select trees for removal, the system applies only to **unmanaged** stands. To accurately project **managed** stands a procedure is needed that will remove trees (simulate cutting) according to prescribed guidelines. This paper describes the procedure (algorithm) used to simulate tree removals recommended in management guides, and indicates some possible uses for the projection system with the addition of the algorithm. FORTRAN subroutines called by the projection system computer program (Hahn and Brand 1979) implement the tree removal algorithm.

DESCRIPTION OF THE SILVICULTURAL ALGORITHM

The algorithm has three major parts. Because ifferent species produce different products, mature at different ages, and grow under different conditions, the guides used by forest managers are species dependent, and are therefore developed for specific cover types. In the projection system a tree list characterizes the portion of the forest of interest (Leary 1979). The first part of the algorithm determines, on the basis of this tree list, the cover type—and thus, the most appropriate guide to use. The selected guide, along with plot values, determine a silvicultural action to be carried out on the tree list. The final part of the algorithm performs the suggested action.

Cover type may be determined by the species with the greatest number of stems, number of dominant and codominant stems, basal area, basal area of dominants and codominants, or some other method. The Society of American Foresters (1954) favors using the number of dominant and codominant stems. The USDA Forest Service (1974) states that cover type should be based on species most fully occupying the area. The latter point indicates that tree size as well as number is important. Since crown class cannot be projected by the model, a classification based on crown position is impractical. To incorporate size into the procedure, basal area rather than total stems was used to determine cover type. An algorithm used by North Central Forest Experiment Station Forest Survey [2] is the basis for determining the type.

SELECTING THE ACTION

In most cases, guides prepared by North Central Forest Experiment Station silviculturists were used to develop the algorithms. Each guide was translated to a diagram that systematically evaluates stand characteristics to reach a final recommendation. The sources for the diagrams are as follows:

Cover type	*Source*
Jack pine	Benzie (1977a)
Red pine	Benzie (1977b)
White pine	USDA Forest Service (1958)
Spruce-fir	USDA Forest Service (1967a)
Black spruce	Johnston (1977a)
Northern white-cedar	Johnston (1977b)
Mixed swamp conifers	Johnston (1977b)
Lowland hardwoods	Godman (personal communication)

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[1] *This work was conducted under a cooperative agreement between the North Central Forest Experiment Station and the College of Forestry, University of Minnesota, St. Paul, Minnesota.*

[2] *Hahn, J. T. Personal communication, 1976.*

Northern hardwoods	Tubbs (1977), Godman (personal communication)
Oak-hickory	Sander (1977)
Aspen-paper birch	Perala (personal communication) USDA Forest Service (1967b)
Understocked stands of above cover types	Benzie (personal communication)

Cover Typing

The purpose of the typing algorithm is to determine the most appropriate cover type and therefore the correct guide for evaluating the tree list. Most tree lists will have a cover type assigned to them that was either determined in the field or by computer analysis. In either case, there are advantages to determining cover type directly from the tree list using a computer algorithm. One is that it will ensure that identical tree lists have the same cover type regardless of their source. It also allows the cover type to change with projected composition changes.

These guides have several advantages over other possible guides. They enjoy rather widespread use in the Lake States and thus it is more likely that region-wide projections will be accurate. They exist for the major timber types in the Lake States, so there is some consistency. And finally, they do not recommend actions so intensive that the model cannot respond to them. For example, fertilization could not be adequately represented by the model because coefficients are not available to predict growth responses to fertilization.

Since management guides are interpreted by foresters in the field, professional judgment can influence the choice of a final action. To use computer methods in determining an action requires decisions using a tree list that does not contain all the information available in the field. As a result, some simplifying assumptions had to be made in constructing algorithms for the various types. These assumptions were checked with silviculturists and appear to be reasonable.

Various actions are suggested by the guides such as "do nothing," "clearcut", "thin", "shelterwood cut", and "remove overtopping trees". The recommended actions are based on site quality, stand characteristics, and cover type. For example, clearcutting is prescribed in a jack pine stand if the site index is 60 and stand age is 65 years. Clearcutting in an aspen stand, however, is prescribed if the stand age is 35 years.

Decision diagrams containing these criteria were produced for each type. Stand and site characteristics are used to guide decisions in the diagrams, which are constructed along the lines of a taxonomic key (fig. 1). For example, assume a jack pine tree list has the following stand characteristics: average stand diameter—5 inches; basal area—100 square feet/acre; site index—50 feet; stand age—35 years; number of trees—500/acre; and BAMIN—55.6 square feet/acre (BAMIN is the amount of basal area needed to achieve full stocking.). The dashed line in figure 1 indicates how a final recommendation would be reached. Since average stand diameter is 5 inches, the bottom branch is followed. Again the bottom branch is taken since the basal area is greater than 55.6 square feet per acre. The site index and age indicate that the top branches are now appropriate, leading to the final recommendation, "do nothing".

A disadvantage of this approach is that it lacks flexibility—only this decision diagram can be used for jack pine. To allow more flexibility, the critical values are read as input variables and can therefore be changed. Figure 2 shows the more general form of the decision diagram for jack pine. Now, by selecting JP(7) as 50 years and the rest equal to zero, jack pine can be managed on a 50-year rotation with no intermediate cutting. The guides can be manipulated to test the effect of different maximum and residual basal areas, different rotation ages, or less intensive management.

Marking

After an action has been selected according to the guide, it must be carried out. The marking part of the algorithm selects trees for removal according to tree characteristics. In the simple case of clearcutting, all live trees are "marked" as cut. For more complicated actions, such as thinning, characteristics must be selected that allow ranking of trees for desirability of removal. Low-preference trees are marked for cutting until the required residual stand is achieved.

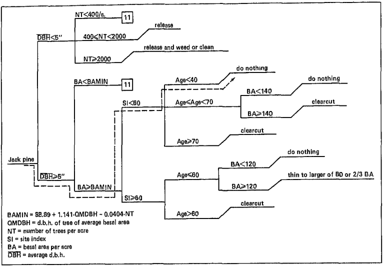

Figure 1.—*Specific jack pine decision diagram.*

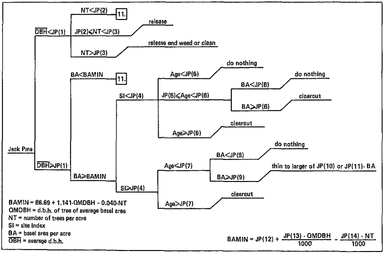

Figure 2.—*Generalized jack pine decison diagram.*

The various removal procedures used to carry out specified actions are listed below:

Removal procedure	*Input needed*
1. rough and rotten	size class to remove
2. small trees, unspecified species	size class to remove four species to leave
3. large trees, unspecified species	size class to remove four species to leave
4. small trees, specified species	size class to remove four species to remove
5. large trees, specified species	size class to remove four species to remove
6. shelterwood cut	size class to remove four species to favor
7. reduce number of trees	number of trees to leave
8. all overtopping	lowest d.b.h. of trees to remove
9. all trees of a certain size and species	size class to remove four species to remove

The removal procedures have been selected to accomplish all the final actions recommended by the management guides. For instance, jack pine stands less than 60 years old with 120 square feet of basal area per acre and a site index greater than 60 should be thinned from below to 80 square feet per acre. The first, second, and fourth procedures will accomplish this action. Rough and rotten trees are removed first, and if the basal area is still too high, the smallest tree of an undesirable species is removed. When all undesirable species are removed and the residual stand has still not been achieved, the smallest trees of desirable species are removed. Marking is discontinued when the residual basal area is achieved.

As with action selection, it is desirable to have the ability to change the marking rules. To permit this flexibility, different hierarchies of removal procedures may be selected. This option permits up to five of the removal procedures listed above to be selected for each recommended action. Returning to the jack pine example—by selecting the first, third, and fifth removal procedures the stand can be thinned in such a way that the largest trees are removed first.

RELATIONSHIP TO THE REST OF THE PROJECTION SYSTEM

Because effective use of the management guide algorithm depends on the rest of the projection system, it is important to present a simplified description of how the pieces interrelate. A tree list (representing a stand) is brought into the system and its cover type and silvicultural action are determined as previously described. The action is carried out and the appropriate trees are coded as cut. One year's growth is determined for the remaining trees and the mortality function applied. Trees are coded as dead if selected by the mortality function. The revised tree list is cycled through the growth and mortality functions to determine annual changes until the desired number of years between cutting cycles has occurred. Cover type, silvicultural action, and trees to mark are again determined using the current tree list. This procedure continues until the tree list has been projected for the desired number of years. (If another stand (tree list) is to be projected, the new tree list is brought into the system and the entire cycle is repeated.)

POSSIBLE USES

Incorporation of the management algorithms into the projection system makes possible several additional uses of the system. Survey information in the form of tree lists, is available on a statewide basis. Applying the projection system to these tree lists would produce timber yields expected from the state if all lands were managed according to the guides. Comparison with present yields would indicate the intensity of management as well as opportunities for increasing timber yields.

Another use for the system would be testing various management guides and marking rules. Actual or contrived tree lists could be projected using different guides, and the yields then compared. This would provide a much faster means of comparing alternatives and help identify those most promising for field testing. By experimenting with marking rules, the effect on timber yields of leaving rough and rotten trees as den trees or favoring different species or size classes could be determined. Comparing these yields with those obtained from the original marking rules would be a meaure of the "cost" of the new marking rule.

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AN IMPLEMENTATION PROGRAM

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The various parts of the generalized forest growth projection system have been described elsewhere (Buchman 1979, Christensen *et al.* 1979, Leary, *et al.* 1979, Hahn and Leary 1979, Leary and Holdaway 1979, Brand 1979). TRES (Timber Resource Evaluation System) is the computer program that combines the parts into a package capable of simulating the growth of trees entered into the system. This paper is not intended to be a detailed user's guide, but rather a general description of the input needed, how the parts are put together, and the output produced.

INPUT

The input consists of a parameter deck and a tree list. A tree list consists of tree species, d.b.h., crown ratio, and quality class for a group of trees. Also required are site index, stand age, and plot size.

Parameter Deck

The parameter deck specifies two groups of input values: user-selected options, and numerical constants needed by each portion of the growth processor.

Numerical constants are used for:

(1) assigning species group codes to individual trees,
(2) crown ratio function coefficients,
(3) silvicultural treatment specifications,
(4) volume equation coefficients,
(5) mortality equation coefficients,
(6) potential growth function coefficients,
(7) growth modifier coefficients,
(8) allocation rule coefficients, and
(9) a species grouping priority matrix (used for combining species when more than three species occur on a plot).

These values are determined from calibrating the model on a specific data base of permanent (remeasured) plots.

Projection related options that may be selected by the user are:

(1) length of projection cycle (years),
(2) number of projection cycles, and
(3) type of growth projection desired (validation, projection, or update).

The number of years the tree list is projected is determined by the number and length of the cycles. The projection is limited to a maximum of 100 years, but it may be in the form of 100 cycles of 1 year each or one cycle of 100 years. The length of the cycle determines how often summaries are made and the tree list examined for management opportunities.

Three types of growth projections can be selected: validation, projection, and update. A validation run uses a tree list with measurements recorded for several different years. The tree list is projected from the initial measurement to the final measurement. Projected and actual measurements are compared to determine the accuracy of the model. A projection run starts with initial conditions and grows the trees for a specified length of time to estimate future conditions. Finally, an update run starts with initial conditions from the past and grows the trees to the present. Using known product removals, the trees that produced those products are removed from the tree list. This update section of the program is currently being developed. An example of an echo of the input for a validation run appears in Appendix A.

Tree List

Regardless of whether the tree list comes from sample plots or is derived (for example, from a stand table), the tree list can be in one of three forms. The first is a list in the format described by Christensen *et al.* (1979). The second is tree information in the unit record format used by the USDA Forest Service Renewable Resources Evaluation (forest survey) projects. And finally, a tree list in a computer-readable format specified by a user may be selected, in which case the user is responsible for supplying the necessary computer code for input and output routines.

PROGRAM ORGANIZATION

Besides providing input/output capability, the program controls when the different parts of the system are used according to the options selected. Table 1 gives a listing of all the subroutine and function names, their calling routines, and a brief statement of their purpose. The program has seven parts:

(1) the main calling program,
(2) a function, VALUE, where all mathematical equations are located,
(3) input subroutines,
(4) a subroutine for aggregating the input tree list,
(5) stand treatment subroutines,
(6) stand growth subroutines, and
(7) output subroutines.

Figures 1 to 7 contain flow charts for each of the seven parts. Figure 1 shows the structure of the main calling routine and figure 2 the structure of the function VALUE.

Figure 3 shows the set of subroutines for reading the input parameters and the tree list. The structure of subroutine READ is shown in figure 3a. It reads input parameters and echo-prints all values. Figure 3b shows the subroutine structure for reading tree lists in master format (Christensen *et al.* 1979); figures 3c and 3d show subroutine structure for reading tree lists in Renewable Resources Evaluation (forest survey) unit record format. The possibility of a user-written subroutine is indicated by figure 3e.

When all input (parameters and tree list) has been read, the tree list is partitioned into species groups and size classes by subroutine FIT (fig. 4), called from the main program TRES (fig. 1).

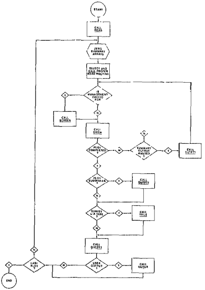

Figure 1.—Main program calling **TRES.**

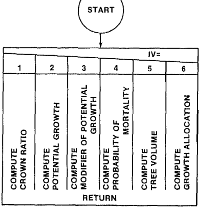
START
IV=
1
2
3
4
5
6
COMPUTE CROWN RATIO
COMPUTE POTENTIAL GROWTH
COMPUTE MODIFIER OF POTENTIAL GROWTH
COMPUTE PROBABILITY OF MORTALITY
COMPUTE TREE VOLUME
COMPUTE GROWTH ALLOCATION
RETURN

Figure 2.—Function **VALUE.**

Table 1.—*Subsystem TRES (Timber Resource Evaluation System) of program FREP (Forest Resources Evaluation Program)*

	Part Function
Program TRES Main calling routine Sets up and initializes arrays, performs housekeeping functions, organizes calling of subroutines	1 Main Program
Function VALUE Called from RMASTR, RSURTP, GROW, ISUM, OUTPTI, and OUTPUT2 Computes values for crown ratio, potential growth, modifier, mortality, volume, and growth allocation functions.	2 Mathematical functions
Subroutine READ Called from TRES Reads user-selected options and numerical constants and echo-prints all values.	3 Input
Subroutine RMASTR Called from TRES Reads data from binary files in MASTER format into arrays TRS and ITRS Initializes array parameters	
Subroutine RSURTP Called from TRES. Reads data from files in forest survey unit record format into arrays TRS and ITRS. Initializes array parameters.	
Subroutine OPENRD Called from RSURTP Unblocks forest survey unit record file and supplies one record at a time to RSURTP Has entry point READIT.	
Subroutine RUSER Called from TRES Supplied by user to read user dat file into arrays TRS and ITRS.	
Subroutine FIT Called from TRES, SCREEN and REMOVE. Classifies trees in arrays TRS-ITRS into projection groups	4 Aggregation of species
Subroutine SCREEN Called from TRES. Screens plots for possible treatment and/or sets up treatment schedule	5 Stand treatments
Subroutine COVTYPE Called from SCREEN Determines cover type using live trees in tree list.	
Subroutine TREAT Called from SCREEN Performs silvicultural treatments prescribed in SCREEN.	
Subroutine GROW Called from TRES. Grows trees, selects trees for mortality, allocates growth to individual trees, updates TRS and ITRS array.	6 Stand growth
Subroutine REMOVE Called from GROW. Removes trees to meet specified annual removals for updates or trees recorded as having been removed by remeasurements.	
Subroutine ISUM Called from GROW, OUTPTI, TRES Summarizes trees into classification groups and computes group totals.	
Subroutine OUTPTI Called from TRES. Outputs summaries for specified points in time.	7. Output
Subroutine TFILE Called from TRES. Outputs updated forest survey unit record file.	
Subroutine OPENWR Called from TFILE. Writes blocked unit record tape Has entry points WRITIT and CLOSWR.	
Subroutine OUTPT2 Called from TRES. Outputs file for validation tests.	
Subroutine OUTUR Called from TRES. Supplied by user to output updated tree list to user specifications.	

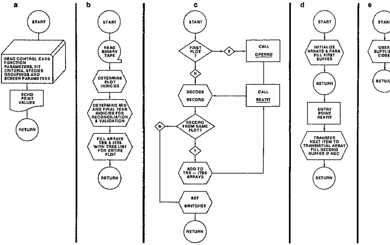
a
START
READ CONTROL CARD FUNCTION PARAMETERS, FIT CRITERIA, SPECIES GROUPINGS AND SCREEN PARAMETERS
ECHO PRINT VALUES
RETURN
b
START
READ BINARY TAPE
DETERMINE PLOT INDICIES
DETERMINE MID AND FINAL YEAR INDICIES FOR RECONCILIATION & VALIDATION
FILL ARRAYS TRS & ITRS WITH TREE LIST FOR ENTIRE PLOT
RETURN
c
START
FIRST PLOT ?
Y
CALL OPENRD
N
DECODE RECORD
CALL REATIT
RECORD FROM SAME PLOT?
N
Y
ADD TO TRS — ITRS ARRAYS
SET SWITCHES
RETURN
d
START
INITIALIZE ARRAYS & PARA FILL FIRST BUFFER
RETURN
ENTRY POINT REATIT
TRANSFER NEXT ITEM TO TRANSMITIAL ARRAY FILL SECOND BUFFER IF NEC
RETURN
e
START
USER SUPPLIED CODE
RETURN

Figure 3.—Subroutines (a) **READ,** *(b)* **RMASTR,** *(c)* **RSURTP,** *(d)* **OPENRD,** *and (e)* **RUSER.**

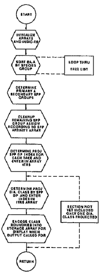
START
INITIALIZE ARRAYS AND INDICIES
SORT DIA BY SPECIES GROUP
LOOP THRU TREE LIST
DETERMINE PRIMARY & SECONDARY SPP GROUPS
CLEANUP REMAINING SPP GROUP ASSIGN ACCORDING TO SPP AFFINITY ARRAY
DETERMINE PROJ SPP OP INDEX FOR EACH TREE AND ENTER IN ARRAY ITRS
DETERMINE PROJ DIA. CLASS BY SPP OP AND ENTER INDEX IN ITRS ARRAY
SECTION NOT YET INCLUDED ONLY ONE DIA. CLASS PROJECTED
ENCODE CLASS BOUNDRIES INTO STORAGE ARRAY FOR DISPLAY WHEN OUTPUT CALLED FOR
RETURN

Figure 4.—Subroutine **FIT.**

Selecting the option to manage the stand as it is being projected, following the management guides of Brand (1979), causes subroutines SCREEN, COVTYPE and TREAT to be called. Figure 5a shows the flow chart of SCREEN, from which COVTYPE and TREAT are called. COVTYPE (fig. 5b), which is also called at the end of a projection by OUTPUT2, determines the cover type; TREAT (fig. 5c) performs the silvicultural treatments selected in SCREEN. Trees selected for removal are designated as cut, and the live trees are again divided into species and diameter groups by a recall of subroutine FIT. This allows the changes in species composition caused by cutting to modify the numerical constants used to project the plot.

The tree list is projected 1 year at a time for the number of years in a cycle and for the number of cycles in the run by subroutine GROW, REMOVE, and ISUM (fig. 6). A plot is then projected through the end of projection period; it cannot be revisited until all plots have been processed. The crown ratio function (Holdaway *et al.* 1979), potential growth function (Hahn and Leary 1979), and modifier function (Leary and Holdaway 1979) are used to determine total growth of each diameter/species projection group (fig. 6a). Individual tree growth is then determined from the growth allocation function (Leary *et al*. 1979). Mortality trees are determined using the mortality function (Buchman 1979) and pseudo-random number generator.

OUTPUT

Four different output types are possible:

1. "Yield table" output containing such variables as basal area and number of mortality trees, surviving trees, basal area and sum of diameters on a per plot and per acre basis, and cut tree volumes on a per acre basis. This output is given by subroutine OUTPT1 (fig 7a) and is available at the end of each cycle as well as at the beginning and end of the projection period. Appendix B shows an example of this form of output.
2. A new Renewable Resources Evaluation (forest survey) unit record file (subroutine TFILE and OPENRD, figs. 7b and 7c respectively).
3. A file for validating the projection system. This is produced by subroutine OUTPT2 (fig. 7d) and has meaning only if two or more measurements have been made on the plot. The file produced by OUTPT2 is ordinarily stored on disk and analyzed by a separate user-written program.
4. A user-designed output file (subroutine OUTUR, fig. 7c).

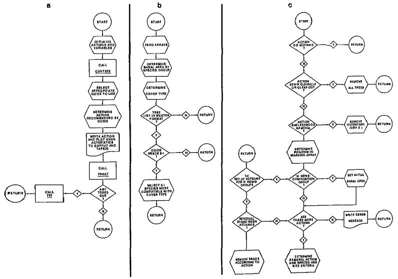
a
START
INITIALIZE ACTIONS AND VARIABLES
CALL COVTYPE
SELECT APPROPRIATE GUIDE TO USE
DETERMINE ACTION RECOMMENDED BY GUIDE
WRITE ACTION AND PLOT CHARACTERISTICS TO OUTPUT AND TAPE20
CALL TREAT
ANY TREES CUT ?
Y
CALL FIT
RETURN
N
RETURN
b
START
ZERO ARRAYS
DETERMINE BASAL AREA BY SPECIES GROUP
DETERMINE COVER TYPE
TREE LIST IN MASTER FORMAT ?
N
RETURN
Y
GUIDE NEEDS SI ?
N
RETURN
Y
SELECT SI SPECIES MOST COMPATIBLE WITH COVER TYPE
RETURN
c
START
ACTION DO NOTHING ?
Y
RETURN
N
ACTION STRIP CLEARCUT OR CLEAR CUT ?
Y
REMOVE ALL TREES
RETURN
N
ACTION SHELTERWOOD REMOVAL ?
Y
REMOVE OVERSTORY (DBH ≥)
RETURN
DETERMINE POSITION IN MARKING ARRAY
RETURN
1st SET OF ACTIONS FOR N HDWD GROUP
Y
N HDWD COMPOSITION GROUP ?
Y
SET INITIAL BASAL AREA
N
Y
RESIDUAL STAND BEEN ATTAINED ?
N
ARE THERE MORE ACTIONS ?
N
WRITE ERROR MESSAGE
RETURN
Y
REMOVE TREES ACCORDING TO ACTION
DETERMINE REMOVAL ACTION AND SPECIES AND SIZE CRITERIA

Figure 5.—Subroutines (a) SCREEN, *(b)* COVTYPE, *and (c)* TREAT.

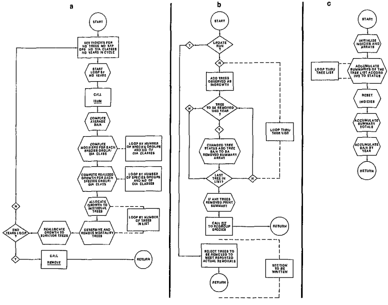
a
START
SET INDICIES FOR NO TREES NO SPP OPS NO DIA. CLASSES NO YEARS IN CYCLE
START LOOP BY NO YEARS
CALL ISUM
COMPUTE AVERAGE BA/A
COMPUTE MODIFIERS FOR EACH SPECIES GROUP/ DIA CLASS
LOOP BY NUMBER OF SPECIES GROUPS AND NO OF DIA CLASSES
COMPUTE REALIZED GROWTH FOR EACH SPECIES GROUP/ DIA CLASS
LOOP BY NUMBER OF SPECIES GROUPS AND NO OF DIA CLASSES
ALLOCATE GROWTH TO INDIVIDUAL TREES
LOOP BY NUMBER OF TREES IN LIST
N
END YEARS LOOP ?
REALLOCATE GROWTH TO SURVIVOR TREES
DETERMINE AND REMOVE MORTALITY TREES
Y
CALL REMOVE
RETURN
b
START
UPDATE RUN ?
Y
N
ADD TREES OBSERVED AS INGROWTH
TREE TO BE REMOVED THIS YEAR ?
N
Y
LOOP THRU TREE LIST
CHANGED TREE STATUS ADD TREE BA/A TO BA REMOVED SUMMARY ARRAY
LAST TREE IN LIST?
N
IF ANY TREES REMOVED PRINT SUMMARY
CALL FIT TO REGROUP SPECIES
RETURN
SELECT TREES TO BE REMOVED TO MEET REPORTED ACTUAL REMOVALS
SECTION TO BE WRITTEN
RETURN
c
START
INITIALIZE INDICIES AND ARRAYS
LOOP THRU TREE LIST
ACCUMULATE SUMMARIES OF THE TREE LIST ACCORDING TO STATUS
RESET INDICIES
ACCUMULATE SUMMARY TOTALS
ACCUMULATE BA/A BY YEAR
RETURN

Figure 6.—Subroutines (a) **GROW,** *(b)* **REMOVE,** *and (c)* **ISUM.**

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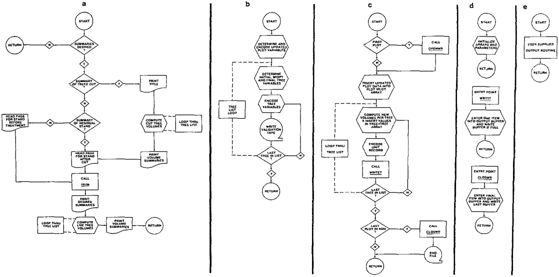

Figure 7.—Subroutines (a) **OUTPT1,** *(b)* **OUTPT2,** *(c)* **TFILE,** *(d)* **OPENWR,** *and (e)* **OUTUR.**

APPENDIX A

```
                      TEST OF UPDATE CHANGES USING CLOQUET FILE

         THIS IS A VALIDATION RUN -    1 PLOTS WILL BE PROCESSED
                   ----------

         DATA WILL BE READ FROM LOGICAL UNIT 11 USING MASTER FORMAT

         OUTPUT WILL CONSIST OF PROPERTY SUMMARIES PLUS USER SPECIFIED TAPE FILE AND    VALIDATION SUMMARIES

         THE PLOT SUMMARIES CALLED FOR AND THEIR ORDER ARE-
            1    2    3    4    5    6    7

         THE PROJECTION WILL CONSIST OF  5 CYCLES OF  2 YEARS EACH

         THE RANDOM NUMBER INITIALIZATION VALUE IS 31 WHICH GIVES A SEED VALUE OF    .6778029E+00

   SPECIES GROUP ASSIGNMENTS ARE-
                          NUMBER SPECIES GROUPS = 31 CODE OTHER SOFTWOODS = 10 CODE OTHER HARDWOODS = 30
     SPP.  SG     SPP.  SG     SPP.  SG     SPP.  SG     SPP.  SG     SPP.  SG     SPP.  SG     SPP.  SG     SPP.  SG     SPP.  SG
       12   5       71   7       94   4       95   6      105   1      125   2      129   3      130   1      241   8      261   9
      314  18      316  14      317  13      318  18      371  16      375  26      402  23      403  23      405  23      407  23
      409  23      541  19      543  11      544  11      741  25      742  12      743  24      746  25      802  20      804  20
      806  22      809  22      823  20      826  20      830  22      832  20      833  21      834  21      837  22      951  17
      972  15      974  15      975  15      977  15      999  31

   SPECIES GROUPS ARE-
    1 JACK PINE      2 RED PINE      3 WHITE PINE    4 WH. SPRUCE    5 BALSAM FIR    6 BLK.SPRUCE    7 TAMARACK       8 N.WH.CEDAR
    9 HEMLOCK       10 OTHER SWDS   11 BLK.GR.ASH   12 COTTONWOOD   13 SIL.MAPLE    14 RED MAPLE    15 ELM          16 YEL.BIRCH
   17 BASSWOOD      18 HARD MAPLE   19 WHITE ASH    20 WHITE OAK    21 S.RED OAK    22 O.RED OAK    23 HICKORY      24 B.T.ASPEN
   25 QUAK.ASPEN    26 PAP.BIRCH    27              28              29              30 OTHER HWDS   31 NONCOMMER.   32 TOTAL

                             PARAMETERS TO COMPUTE C.R. AS A FUNCTION OF AVERAGE BASAL AREA
 S.G.  PARAMETER1  PARAMETER2  PARAMETER3  PARAMETER4  PARAMETER5    S.G.  PARAMETER1  PARAMETER2  PARAMETER3  PARAMETER4  PARAMETER5
   1        0        8.000     .1870E-01    2.000     .8490E-01       2        0        8.000     .1270E-01    2.000     .5040E-01
   3        0        8.000     .6000E-02    2.000     .1399           4        0        8.000     .5800E-02    2.000     .8150E-01
   5        0        8.000     .5800E-02    2.000     .8150E-01       6        0        8.000     .3900E-02    2.000     .1045
   7        0        8.000     .3900E-02    2.000     .1045           8        0        8.000     .2130E-01    2.000     .5500E-02
   9        0        8.000     .2130E-01    2.000     .5500E-02      10        0        8.000     .2130E-01    2.000     .5500E-02
  11        0        3.410         0           0          0          12   -.1860E-01   6.910         0           0      .4760E-01
  13   -.1860E-01    6.910         0           0      .4760E-01      14   -.1860E-01   6.910         0           0      .4760E-01
  15        0        3.040         0           0          0          16   -.2220E-01   6.019         0           0      .2640E-01
  17        0        3.000         0           0          0          18   -.6400E-02   4.707         0           0      .2820E-01
  19        0        3.410         0           0          0          20   -.8000E-02   4.728         0           0      .3810E-01
  21   -.8000E-02    4.728         0           0      .3810E-01      22   -.8000E-02   4.728         0           0      .3810E-01
  23   -.8000E-02    4.728         0           0      .3810E-01      24   -.7000E-02   4.298         0           0      .2300E-01
  25   -.7000E-02    4.298         0           0      .2300E-01      26   -.7000E-02   4.298         0           0      .2300E-01
  27   -.7000E-02    4.298         0           0      .2300E-01      28   -.7000E-02   4.298         0           0      .2300E-01
  29   -.7000E-02    4.298         0           0      .2300E-01      30   -.7000E-02   4.298         0           0      .2300E-01
  31   -.7000E-02    4.298         0           0      .2300E-01
```

```
FOREST TYPES FOR THIS RUN ARE-
   1 JACK PINE                2 RED PINE                3 WHITE PINE              11 SPRUCE-FIR               12 BLACK SPRUCE
  14 N. WHITE CEDAR          15 TAMARACK               50 OAK-HICKORY            70 LOWLAND HARDWOODS        80 NORTHERN HARDWOODS
  91 ASPEN                   92 PAPER BIRCH            90 ASPEN-BIRCH            99 NON-STOCKED              -0


                                CRITICAL VALUES FOR DECISION VARIABLES

                                VARIABLE POSITION IN SILVICULTURAL GUIDE
  TYPE        1/212/22  3/-3  4/24  5/25  6/26  7/27  8/28  9/29 10/30   11    12    13    14    15    16    17    18    19    20

JACK PINE      5.0 400.020 6.0  60.0  40.0  70.0  60.0 140.0 120.0  80.0    .7  68.91141.0  40.4

RED PINE       5.0 400.0   2.0   9.0  15.0  20.02000.0 160.0 160.0 140.0 120.0 140.0 180.0 200.0  90.0    .5 120.0    .5 150.0  86.6
            1328.0  85.9

WHITE PINE     5.0 400.0  70.0  40.0 120.0  40.0  40.0  40.0  40.0  60.0     0     0     0     0 117.92934.0 147.8

SPRUCE-FIR    40.0  50.0  45.0  20.0  40.0 100.0  45.0  35.0 140.0  75.0 100.0  55.0

B. SPRUCE     25.0 147.6 571.9   3.3  90.0-175.4   2.6

SWAMP CON.    25.0  36.0 175.0  50.0  60.0  80.0  90.0 100.0  50.0  60.0  80.0  60.0  60.0

LOW. HDWD.    55.0  75.0  -0.0 100.0 100.0  65.0  65.0  75.0

N. HDWD.       2.0   4.650 0.01000.0  30.0   9.6  20.0  16.0  20.0  70.0 110.0  75.0 160.0 150.0  80.0  80.0  85.0 120.0  70.0  70.0
              70.0     0  35.0  15.0

OAK-HICK.     55.0  75.0  -0.0 130.0  60.0  60.0  50.0  70.0    .5    .5    .5  10.0  10.0  10.0  16.0  20.0  24.0  10.0  25.0  10.0
              25.0  10.0  25.0  15.0  15.0  15.0  47.22443.0  20.5  65.0 887.0  54.3

ASPEN         40.0  35.0  25.0  55.0  55.0  75.0 100.0  35.0  60.0  50.0  75.0  45.0  50.0  75.0  85.0 110.0  90.0   6.0  55.0  80.0
               6.0

P. BIRCH      40.0  35.0  25.0  55.0  55.0  75.0 100.0  35.0  60.0  50.0  75.0  45.0  50.0  75.0  85.0 110.0  90.0   6.0  55.0  80.0
               6.0

N.W. CEDAR    25.0  36.0 170.0  50.0  60.0 150.0  50.0  60.0 130.0 200.0 200.0 150.0  60.0 150.0  60.0

TAMARACK        -0    -0

UNDERSTKD.    20.0  15.0
```

```
                         VOLUME REGRESSION EQUATION PARAMETERS

         NUMBER OF EQUATIONS =  6 FOR WISCONSIN NORTHEASTERN UNIT 1  CUBIC FOOT VOLUME TREE CLASS 10

 SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 EQUATION NO.   1  1  1  2  2  2  2  2  2  2  4  5  4  4  6  4  6  4  4  3  3  3  3  5  5  5  6  6  6  6  6

         EQU.        PARA.1          PARA.2          PARA.3          PARA.4          PARA.5
          1        36048.68       .4024466E-04     1.173027       -.1517975         -0
          2        385.4175       .1070582E-03     1.350854       -.2868318         -0
          3        1317.764       .4733759E-03     1.462066       -.1206417         -0
          4        25157.79       .4449758E-04     1.177665       -.1559213         -0
          5        562.9609       .7814853E-03     1.549974       -.1390638         -0
          6        537.5624       .5394414E-03     1.295829       -.2497832         -0

         NUMBER OF EQUATIONS = 16 FOR WISCONSIN NORTHEASTERN UNIT 1  CUBIC FOOT VOLUME TREE CLASS 20

 SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 EQUATION NO.   3  2  1  4  5  4  6  6  6  6 12 12 11 11 16  9 15 10 12  7  7  8  8 13 14 13 16 16 16 16 16

         EQU.        PARA.1          PARA.2          PARA.3          PARA.4          PARA.5
          1        1199.625       .4136829E-03     1.268860       -.2083986         -0
          2        127.1188       .3939387E-03     1.810027       -.1883882         -0
          3        63.06930       .1042520E-02     1.888101       -.2019203         -0
          4        573.3734       .4495926E-03     1.571015       -.1433549         -0
          5        591.3112       .4327938E-03     1.563568       -.1418235         -0
          6        565.4679       .4310511E-03     1.290465       -.2412149         -0
          7        777.4550       .8723104E-03     1.468083       -.1239891         -0
          8        397.7769       .1007421E-02     1.527054       -.1354224         -0
          9        118.3166       .1889391E-03     1.831568       -.1925592         -0
         10        246.6457       .4292254E-03     1.666395       -.1624640         -0
         11        714.8841       .6664804E-03     1.494599       -.1290761         -0
         12        11794.36       .9004474E-04     1.183965       -.1608640         -0
         13        144.3193       .1482680E-02     1.710300       -.1696353         -0
         14        64.81606       .1126818E-02     1.897307       -.2034324         -0
         15        773.7419       .5591250E-03     1.505967       -.1306789         -0
         16        559.6307       .6039853E-03     1.288580       -.2364948         -0

         NUMBER OF EQUATIONS =  6 FOR WISCONSIN NORTHEASTERN UNIT 1  CUBIC FOOT VOLUME TREE CLASS 30

 SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 EQUATION NO.   1  1  1  2  2  2  2  2  2  2  4  5  4  4  6  4  6  4  4  3  3  3  3  5  5  5  6  6  6  6  6

         EQU.        PARA.1          PARA.2          PARA.3          PARA.4          PARA.5
          1        627.0185       .3466743E-03     1.498787       -.1265395         -0
          2        10276.78       .2696920E-04     1.206486       -.1784514         -0
          3        114.7355       .9698185E-03     1.702548       -.1670065         -0
          4        265.0294       .1087531E-02     1.623151       -.1333784         -0
          5        3011.090       .9405998E-04     1.446883       -.1156433         -0
          6        101425.5       .3047962E-05     1.383356       -.7984840E-01     -0
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                                   VOLUME REGRESSION EQUATION PARAMETERS

          NUMBER OF EQUATIONS =  5 FOR WISCONSIN NORTHEASTERN UNIT 1  CUBIC FOOT VOLUME TREE CLASS 40

SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
EQUATION NO.   1  1  1  1  1  1  1  1  1  1  3  4  3  3  5  3  5  3  3  2  2  2  2  4  4  4  5  5  5  5  5

            EQUA.        PARA.1          PARA.2          PARA.3          PARA.4          PARA.5
              1        812.3681      .4154177E-04       1.478718       -.1223292          -0
              2        133.2531      .5246846E-01       1.252944       -.2518464E-01      -0
              3        47.71806      .3359426E-02       1.962034       -.1313597          -0
              4        4846.439      .4103629E-04       1.390071       -.1031025          -0
              5        329.7758      .4718038E-02       1.364578       -.9341189E-01      -0

          NUMBER OF EQUATIONS =  6 FOR WISCONSIN NORTHEASTERN UNIT 1  BOARD FOOT VOLUME TREE CLASS 10

SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
EQUATION NO.   1  1  1  2  2  2  2  2  2  2  4  5  4  4  6  4  6  4  4  3  3  3  3  5  5  5  6  6  6  6  6

            EQUA.        PARA.1          PARA.2          PARA.3          PARA.4          PARA.5
              1        257231.5      .2784675E-04       1.171308       -.1504486          -0
              2        5683.337      .3319345E-03       1.275705       -.2313738          -0
              3        42344.65      .1357544E-03       1.197160       -.1710607          -0
              4        106680.1      .5604898E-04       1.182841       -.1596589          -0
              5        11165.41      .3651559E-03       1.237404       -.2041724          -0
              6        3083.508      .5270820E-03       1.293385       -.2494871          -0

          NUMBER OF EQUATIONS = 13 FOR WISCONSIN NORTHEASTERN UNIT 1  BOARD FOOT VOLUME TREE CLASS 20

SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
EQUATION NO.   3  2  1  4  5  4  7  7  6  7 11 12 11 11 13  9 13 10 11  8  8  8  8 12 12 12 13 13 13 13 13

            EQUA.        PARA.1          PARA.2          PARA.3          PARA.4          PARA.5
              1        11183.54      .2437308E-03       1.242120       -.2037176          -0
              2        4302.868      .1037676E-02       1.253822       -.2153252          -0
              3        13971.63      .2434394E-03       1.217312       -.1889183          -0
              4        769248.0      .4628491E-05       1.170987       -.1501769          -0
              5        527215.0      .6197405E-05       1.173399       -.1522792          -0
              6        6558.332      .3089813E-03       1.256708       -.2172892          -0
              7        895.7112      .3537490E-03       1.363223       -.2990378          -0
              8        17587.02      .5285970E-03       1.185383       -.1617882          -0
              9        1352.299      .5313031E-03       1.677255       -.1443074          -0
             10        1438.785      .2009731E-03       1.347552       -.2837179          -0
             11        851.1028      .3350519E-02       1.321802       -.2657067          -0
             12        4977.429      .4298018E-03       1.260356       -.2211002          -0
             13        3921.671      .6729626E-03       1.264172       -.2211569          -0

          NUMBER OF EQUATIONS =  2 FOR WISCONSIN NORTHEASTERN UNIT 1  BOARD FOOT VOLUME TREE CLASS 31

SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
EQUATION NO.   1  1  1  1  1  1  1  1  1  1  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2

            EQUA.        PARA.1          PARA.2          PARA.3          PARA.4          PARA.5
              1        730020.7      .7777858E-05       1.111671       -.1306388          -0
              2        528516.2      .2979013E-04       1.090030       -.9087071E-01      -0
```

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                              VOLUME REGRESSION EQUATION PARAMETERS

          NUMBE  OF EQUATIONS =  6 FOR WISCONSIN NORTHEASTERN UNIT 1  CUBIC VOLUME IN SAWLOG PORTION T.C. 10
SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
EQUATION NO.   1  1  1  2  2  2  2  2  2  2  4  5  4  4  6  4  6  4  4  3  3  3  3  5  5  5  6  6  6  6  6

          EQU..        PARA.1           PARA.2           PARA.3           PARA.4           PARA.5
            1        3464.167      .2459303E-03        1.216782        -.1865286            -0
            2        430.2449      .1091619E-03        1.485718        -.2090047            -0
            3        12966.89      .9830624E-04        1.181643        -.1592031            -0
            4        15909.84      .9668812E-04        1.175707        -.1542967            -0
            5        2546.492      .3029995E-03        1.227451        -.1948611            -0
            6        407.4863      .5449546E-03        1.310683        -.2574237            -0

          NUMBE  OF EQUATIONS = 13 FOR WISCONSIN NORTHEASTERN UNIT 1  CUBIC VOLUME IN SAWLOG PORTION T.C. 20
SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
EQUATION NO.   3  2  1  4  5  4  7  7  6  7 11 12 11 11 13  9 13 10 11  8  8  8  8 12 12 12 13 13 13 13 13

          EQU..        PARA.1           PARA.2           PARA.3           PARA.4           PARA.5
            1        1376.083      .4558337E-03        1.242193        -.2061520            -0
            2        3477.933      .5337996E-03        1.191640        -.1672472            -0
            3        760.2680      .5237971E-03        1.258388        -.2185038            -0
            4        9547.071      .4612539E-04        1.210569        -.1816787            -0
            5        8077.398      .3716047E-04        1.221318        -.1900312            -0
            6        961.1018      .6550633E-03        1.248353        -.2110187            -0
            7        1856.803      .2003224E-03        1.235392        -.2009995            -0
           11        2341.854      .7628486E-03        1.194051        -.1691050            -0
           12        686.4042      .5122937E-03        1.269858        -.2279520            -0
           13        1239.743      .7069922E-03        1.231405        -.1980232            -0

          NUMBE  OF EQUATIONS =  2 FOR WISCONSIN NORTHEASTERN UNIT 1  CUBIC VOLUME IN SAWLOG PORTION T.C. 31
SPECIES GROUP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
EQUATION NO.   1  1  1  1  1  1  1  1  1  1  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2  2

          EQU..        PARA.1           PARA.2           PARA.3           PARA.4           PARA.5
            1        3160.909      .5497128E-04        1.232636        -.1987996            -0
            2        122.0531      .9674253E-04        1.480889        -.2655163            -0
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                                              MORTALITY EQUATION ASSIGNMENTS AND PARAMETERS
SPECIES GP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 EQUATION   1  2  3  4  5  6  7  8  9  2 10 11 11 11 12 13 14 15 16 17 18 19 20 21 22 23 24 24 24 24 25
          EQ.NO.     PARAMETER1        PARAMETER2        PARAMETER3        PARAMETER4        PARAMETER5
             1        .3676700          60.17800          1.072700        .5020000E-02        .1222000
             2        1.084300          11.01200          .5886600        .2000000E-03        .4850000
             3        2.350700          33.60900          1.008800        .2010000E-02               0
             4        2.880600          30.77400          1.230900        .3960000E-02               0
             5        2.697600          30.55000          .8996300        .4000000E-02       -.2306900
             6        2.877400          432.9500          1.857600        .2350000E-02               0
             7        .3676700          60.17800          1.072700        .5020000E-02        .1222000
             8        2.877400          432.9500          1.857600        .2350000E-02               0
             9        2.823300          89.56000          1.096600        .3200000E-02               0
            10        1.487500          13.33800          .5429000        .1800000E-03               0
            11        2.240500          56.59400          1.039200        .4940000E-02               0
            12        1.988900          44.54100          1.096500        .8990000E-02               0
            13        2.670500          63.14800          1.381900        .3820000E-02               0
            14       -.5861200          101.8700          1.649300        .4890000E-02        .6839800
            15        .8150000          66.93700          1.593400        .4350000E-02        .7500000
            16        1.782100          232.3100          1.726700        .2220000E-02               0
            17        3.694600          95.63700          1.451700        .1880000E-02               0
            18        2.309200          6999.800          2.661500        .4680000E-02               0
            19       -1.149200          9.108700          2.455200               0            .3500000
            20        3.062600          63.49500          1.049800        .1340000E-02               0
            21        .8805000          56.59300          1.215400        .1288000E-01               0
            22       -.7855000E-01      213.6300          1.712600        .2276000E-01        .2581200
            23        2.180800          11.62600          .4750000        .2900000E-03               0
            24        2.233900          29.62900          1.342700        .7480000E-02               0
            25        2.358100          113.0600          .9993000        .1923000E-01               0


                                              POTENTIAL GROWTH EQUATION ASSIGNMENTS AND PARAMETERS
SPECIES GP  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
 EQUATION   1  2  3  4  5  6  7  8  9  1 10 11 11 11 12 13 14 15 16 17 18 19 20 21 22 23 24 24 24 24 24
          EQ.NO.     PARAMETER1        PARAMETER2        PARAMETER3        PARAMETER4        PARAMETER5
             1        .1606200       -.9000000E-05       3.624500        .4000000E-04        1.000000
             2        .9446000E-01   -.1200000E-03       2.059600        .3500000E-03        .2422500
             3        .2557800       -.8800000E-03       1.726300        .4000000E-04        1.000000
             4        .1705600       -.1451600E-01       1.066000        .5200000E-03        .2729800
             5        .1220000       -.8000000E-03       1.989000        .6000000E-04        1.000000
             6        .1071300       -.1070000E-02       2.001700        .6000000E-04        .9112700
             7        .1114700       -.1000000E-04       3.068500        .3000000E-04        1.000000
             8        .1340300       -.1000000E-05       3.688000        .2000000E-04        1.000000
             9        .1687200       -.3000000E-06       3.573800        .1000000E-04        1.000000
            10        .5880700E-01   -.6000000E-08       5.055900        .2400000E-03        .3155300
            11        .1094800       -.4000000E-04       2.222600        .5000000E-03        .6626000E-01
            12        .2849600       -.1820000E-02       1.529700        .3000000E-04        1.000000
            13        .1515500       -.3000000E-05       3.310400        .7000000E-04        .5730200
            14        .2540200       -.4000000E-05       2.939600        .1000000E-04        1.000000
            15        .1877200       -.7000000E-05       2.583900        .2800000E-03        .8385000E-01
            16        .2116700       -.3000000E-05       3.313100        .1000000E-04        .1243000
            17        .1265400       -.4000000E-05       2.753800        .5000000E-04        .6208600
            18        .1553500       -.1600000E-06       3.536700        .1800000E-03        .2590000
            19        .1735800       -.7600000E-04       2.445100        .3000000E-04        .9522200
            20        .1647100       -.2161000E-02       1.394900        .4000000E-04        .7162800
            21        .2349000       -.9420000E-02       1.104100        .3600000E-03        .1538600
            22        .2164500       -.9100000E-04       2.603000        .4400000E-04        1.000000
            23        .1097100       -.3200000E-03       2.023600        .3400000E-03        .2128800
                      .3751000       -.2479000E-01       .9628800        .4000000E-04        1.000000
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[illegible] PARAMETER OF BETA FUNCTION AND SPECIES INTERACTION ARRAY

SPECIES GROUP	SPECIES GROUP 1,12,23	2,13,24	3,14,25	4,15,26	5,16,27	6,17,28	7,18,29	8,19,30	9,20,31	10,21	11,22
1	-112.0 1.000 1.000	.4830 1.000 1.000	1.000 1.000 1.355	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
2	2.757 1.000 1.000	-121.6 1.000 1.000	1.192 1.000 2.261	1.000 1.000 3.545	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
3	1.000 1.000 1.000	1.098 1.000 1.000	-117.4 1.000 .6620	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.432 1.000	1.000 .2860	1.000 1.000
4	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	-124.6 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
5	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 2.327	1.000 1.000 1.000	-86.29 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	.2860 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
6	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 .2860	1.000 1.000 1.000	1.000 1.000 1.000	-124.6 1.000 1.000	3.523 1.000 1.000	3.066 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
7	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	.2840 1.000 1.000	-52.34 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
8	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	3.500 1.000 1.000	.3630 1.000 1.000	1.000 1.000 1.000	-155.5 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
9	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 .4570 1.000	1.000 1.000 1.000	1.000 .9770 1.000	1.000 1.000 1.000	-117.4 1.000 1.000	1.000 1.000	1.000 1.000
10	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	-117.4 1.000	1.000 1.000
11	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	-73.59 1.000
12	1.000 -68.80 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
13	1.000 1.000 1.000	1.000 -68.80 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000
14	1.000 1.000 1.000	1.000 1.000 1.000	1.000 -68.80 .5740	1.000 1.000 2.677	1.000 1.205 1.000	1.000 1.000 1.000	1.000 1.186 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 .5890	1.000 1.000
15	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.441	1.000 -73.59 1.000	1.000 1.000 1.000	1.000 2.010 1.000	1.000 .8240 1.000	1.000 1.000 1.000	1.000 .2930 1.000	1.000 .6040	1.000 1.000
16	1.000 1.000 1.000	1.000 1.000 1.000	1.000 .8590 1.000	1.000 1.000 1.000	1.000 -72.83 1.000	1.000 1.000 1.000	1.000 1.330 1.000	1.000 1.000 1.000	2.189 1.000 1.000	1.000 1.000	1.000 1.000
17	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 .7000 1.000	1.000 1.000 1.000	1.000 -72.83 1.000	1.000 1.317 1.000	1.000 1.000 1.000	1.000 1.000 1.000	1.000 1.000	1.000 1.000

18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.598	1.000	1.000
	1.000	1.000	1.571	1.431	.7500	.9760	-73.59	.9650	1.000	.6480	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.589	-73.59	1.000	2.346	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		
20	1.000	1.000	.7760	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.800
	1.000	1.000	1.000	3.414	1.000	1.000	1.000	1.000	-72.83	1.601	3.546
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		
21	1.000	1.000	3.500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	2.321	2.090	1.000	1.000	2.357	.4270	.9770	-72.83	.8930
	1.000	2.497	2.324	1.000	1.000	1.000	1.000	1.000	1.000		
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.800	1.000	1.000	1.000	.2820	1.120	-72.83
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	-72.83	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.4470	1.000
	1.000	-117.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000		
25	.8290	.4550	1.783	1.000	.5810	3.500	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.829	.8800	1.000	1.000	1.000	1.000	1.000	.4990	1.000
	1.000	1.000	-116.9	3.500	1.000	1.000	1.000	1.000	1.000		
26	1.000	.2820	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	.3890	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	.2860	-117.4	1.000	1.000	1.000	1.000	1.000		
27	1.000	1.000	1.000	1.000	1.000	1.800	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	-72.83	1.000	1.000	1.000	1.000		
28	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	-72.83	1.000	1.000	1.000		
29	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	-72.83	1.000	1.000		
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	-72.83	1.000		
31	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	-72.83		

SECOND AND THIRD PARAMETERS FOR BETA FUNCTION

					SPECIES GROUP						
PARAM	1,12,23	2,13,24	3,14,25	4,15,26	5,16,27	6,17,28	7,18,29	8,19,30	9,20,31	10,21	11,22
2	-.4638	-.4462	-.1261	-.3652	-.4923	-.3652	-.4441	-.2230	-.1261	-.1261	-.1902
	-.1434	-.1434	-.1434	-.1902	-.1303	-.1303	-.1902	-.1902	-.1303	-.1303	-.1303
	-.1303	-.1261	-.1366	-.1261	-.1303	-.1303	-.1303	-.1303	-.1303		
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		

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                                   ON RULE EQUATION ASSIGNMENT AND PARAMETERS
                   PARAMETERS 1-3 ARE USED TO DETERMINE THE VALUE OF -A- AND PARAMETERS 4-7 FOR THE VALUE OF -B-
 SPECIES GP   1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
  EQUATION    1  3  5  7  9 11 11 13  3  1 19 19 19 19 19 19 19 15 19 19 19 19 19 17 17 17 19 19 19 19 19

           EQUA.      PARA.1      PARA.2      PARA.3      PARA.4      PARA.5      PARA.6      PARA.7
             1     -.6295E-01  -.7303E-03   .7711E-05   .1000E+00  -.4000E-02   .1500E+03   .7400E+00
             2     -.6295E-01  -.7303E-03   .7711E-05   .1000E+00  -.4000E-02   .1500E+03   .7400E+00
             3     -.6431E-01  -.9678E-03   .5162E-05   .2000E+00  -.9447E-02   .3000E+03   .4102E+01
             4     -.8437E-01  -.5417E-03   .3802E-05   .1500E+00  -.5585E-02   .3000E+03   .2139E+01
             5     -.7361E-01  -.3537E-03   .1422E-05   .1000E+00  -.7298E-03   .2500E+03   .1759E+00
             6     -.7361E-01  -.3537E-03   .1422E-05   .1000E+00  -.7298E-03   .2500E+03   .1759E+00
             7     -.1111E+00  -.1006E-03   .3391E-05   .1000E+00  -.1799E-01   .2500E+03   .8264E+01
             8     -.1111E+00  -.1006E-03   .3391E-05   .1000E+00  -.1799E-01   .2500E+03   .8264E+01
             9     -.8707E-01  -.2422E-03   .2626E-05   .1000E+00  -.1720E-02   .2500E+03   .3479E+00
            10     -.8707E-01  -.2422E-03   .2626E-05   .1000E+00  -.1720E-02   .2500E+03   .3479E+00
            11     -.1101E+00   .6208E-04  -.7802E-07   .1000E+00  -.3190E-02   .2500E+03   .3756E+00
            12     -.1101E+00   .6208E-04  -.7802E-07   .1000E+00  -.3190E-02   .2500E+03   .3756E+00
            13     -.1536E+00   .5834E-03  -.1198E-05   .1000E+00  -.2018E-02   .4000E+03   .3724E+00
            14     -.1536E+00   .5834E-03  -.1198E-05   .1000E+00  -.2018E-02   .4000E+03   .3724E+00
            15     -.8396E-01  -.2447E-03   .2451E-05   .1000E+00  -.6177E-02   .2500E+03   .8322E+00
            16     -.8396E-01  -.2447E-03   .2451E-05   .1000E+00  -.6177E-02   .2500E+03   .8322E+00
            17     -.8195E-01  -.2475E-03   .2478E-05   .1000E+00  -.3471E-02   .2500E+03   .7540E+00
            18     -.8195E-01  -.2475E-03   .2478E-05   .1000E+00  -.3471E-02   .2500E+03   .7540E+00
            19     -.3939E-01  -.1959E-02   .1513E-04   .1000E+00  -.2297E-01   .2000E+03   .6384E+01
            20     -.3939E-01  -.1959E-02   .1513E-04   .1000E+00  -.2297E-01   .2000E+03   .6384E+01

                                   SPECIES ASSIGNMENT PRIORITY ARRAY
      1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
  1  -1  8 18 11  3  1  2  4 19 25  6 23 24 16 17 20 14 13 15 21 22  9  5 10 12  7  0  0  0 26 27
  2  21 -1 15  5  2 22  8  1 16 23  9 24 25 13 14 17 11 10 12 18 19  3 20  4  7  6  0  0  0 26 27
  3  21 14 -1 11 16 22 18 15  2 23 19 24 25  3  1  4  7  9  5  6  8 13 20 12 10 17  0  0  0 26 27
  4  20  6 13 -1 10 22 14  8 15 23 16 24 25  9 11 17  5  3  7 19 21  4 18  2  1 12  0  0  0 26 27
  5   5  2 18  4 -1  6  3  1 19 23  9 24 25 16 17 20 14 13 15 21 22  8 12 10 11  7  0  0  0 26 27
  6   1  5 18  9  3 -1  2  4 19 23  7 24 25 16 17 20 14 13 15 21 22 10  6 11 12  8  0  0  0 26 27
  7   2  5 18  6  1  4 -1  3 19 23  8 24 25 16 17 20 14 13 15 21 22 10  9 11 12  7  0  0  0 26 27
  8   5  1 18  3  2  6  4 -1 19 23  8 24 25 16 17 20 14 13 15 21 22  9 10 11 12  7  0  0  0 26 27
  9  21 14  1 11 16 22 18 15 -1 23 19 24 25  6  3  2  8  9  7  4  5 13 20 12 10 17  0  0  0 26 27
 10   1  9 19 12  4  2  3  5 26 -1  7 24 25 17 18 21 15 14 16 22 23 10  6 11 13  8  0  0  0 26 27
 11  11 15 19 16 12 13  9 14 20 27 -1 24 25  8 18 21  6  5  7 22 23  2 10  3  4  1  0  0  0 26 27
 12  24 17 14 16 19 25 21 18 15 27 22 -1  2  1  4 11  5  6  3 12 13 10 23  9  8 20  0  0  0  7 26
 13  24 17 14 16 19 25 21 18 15 27 22  2 -1  1  4 11  5  6  3 12 13 10 23  9  8 20  0  0  0  7 26
 14  24 17 14 16 19 25 21 18 15 27 22 13 12 -1  2  9  3  4  1 10 11  8 23  7  6 20  0  0  0  5 26
 15  24 17 14 16 19 25 21 18 15 26 22 12 13  2 -1  4  5  7  3  6  8 11 23 10  9 20  0  0  0  1 27
 16  24 17 14 16 19 25 21 18 14 26 22  6  7  5  3 -1  9 10  8  1  2 13 23 12 11 20  0  0  0  4 27
 17  24 16 15 14 19 25 21 18 17 26 22 12 13  4  6  9 -1  1  2 10 11  8 23  7  3 20  0  0  0  5 27
 18  24 16 15 14 19 25 21 18 17 26 22 12 13  4  6  9  1 -1  2 10 11  8 23  7  3 20  0  0  0  5 27
 19  24 16 15 14 19 25 21 18 17 26 22 12 13  2  6  9  1  3 -1 10 11  8 23  7  5 20  0  0  0  4 27
 20  24 17 15 16 19 25 21 18 14 26 22 12 13  5  3  2  7  8  6 -1  1 11 23 10  9 20  0  0  0  4 27
 21  24 17 15 16 19 25 21 18 14 26 22 12 13  5  3  2  7  8  6  1 -1 11 23 10  9 20  0  0  0  4 27
 22  19 14 21 15 17 26 18 16 22 26  7  9 10  8 10 23  5  3  6 24 25 -1 11  1  2  4  0  0  0 12 27
 23  11 15 19 16 12 13 10 14 20 27  1 24 25  9 18 21  7  6  8 22 23  3 -1  4  5  2  0  0  0 17 26
 24  19 14 21 15 17 26 18 16 22 26  7  9 10  8 13 23  5  3  6 24 25  1 11 -1  2  4  0  0  0 12 27
 25  24 18 21 17 20 25 23 19 22 26 10 15 16  6  8 11  3  1  5 12 14  4 13  2 -1  9  0  0  0  7 27
 26  18 17 21 20 19 19 15 16 22 26  1 13 14  9 12 23  7  6  8 24 25  3  2  4  5 -1  0  0  0 11 27
 27   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 28   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 29   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 30  24 17 14 16 19 25 21 18 15 26 22 12 13  1  2  6  4  5  3  8  9 11 23 10  7 20  0  0  0 -1 27
 31   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1
```

OUTPUT RECORD FOR TEST OF UPDATE CHANGES USING CLOQUET FILE

APPENDIX B

```
                          TEST OF UPDATE CHANGES USING CLOQUET FILE
 ******** PLOT SUMMARIES AT THE END OF 1959 PLOT IDENT.     1   ********

     NUMBER OF MORTALITY TREES PER ACRE
        SPECIES                       DIAMETER CLASS
         GROUP            ALL    TOTAL
       JACK PINE            0        0
       PAP.BIRCH            0        0
       BLK.SPRUCE           0        0
       TOTAL                0        0

     DIAMETER SUMS PER ACRE
        SPECIES                       DIAMETER CLASS
         GROUP            ALL    TOTAL
       JACK PINE        372.4    372.4
       PAP.BIRCH        125.3    125.3
       BLK.SPRUCE           0        0
       TOTAL            497.7    497.7

     BASAL AREA OF MORTALITY TREES PER ACRE
        SPECIES                       DIAMETER CLASS
         GROUP            ALL    TOTAL
       JACK PINE            0        0
       PAP.BIRCH            0        0
       BLK.SPRUCE           0        0
       TOTAL                0        0

     BASAL AREA ON PLOT
        SPECIES                       DIAMETER CLASS
         GROUP            ALL    TOTAL
       JACK PINE          3.1      3.1
       PAP.BIRCH           .6       .6
       BLK.SPRUCE           0        0
       TOTAL              3.7      3.7

     NUMBER OF TREES PER ACRE
        SPECIES                       DIAMETER CLASS
         GROUP            ALL    TOTAL
       JACK PINE         35.0     35.0
       PAP.BIRCH         21.0     21.0
       BLK.SPRUCE           0        0
       TOTAL             56.0     56.0

     BASAL AREA PER ACRE
        SPECIES                       DIAMETER CLASS
         GROUP            ALL    TOTAL
       JACK PINE         22.0     22.0
       PAP.BIRCH          4.1      4.1
       BLK.SPRUCE           0        0
       TOTAL             26.0     26.0

     NUMBER OF TREES ON PLOT
        SPECIES                       DIAMETER CLASS.
         GROUP            ALL    TOTAL
       JACK PINE          5.0      5.0
       PAP.BIRCH          3.0      3.0
       BLK.SPRUCE           0        0
       TOTAL              8.0      8.0

                  PRESENT STAND VOLUMES
              SPP.GP.   CUFT/A.   CORDS/A.    BDFT/A.
         JACK PINE       389.50       4.93    1860.35
         PAP.BIRCH        68.91        .87          0
         TOTAL           458.41       5.80    1860.35
```

```
                                              SUMMARY STATISTICS

PLOT     1              FOREST TYPE     1           SI   61          AGE   41              INIT. LIVE TREES  15                INIT. BA   26
                                                                                           FINAL LIVE TREES  13                FINAL BA    0
**************************************************************************************************************************************************
                 DBH    BA EXP T  EXP   TREE   TREE        CUR.    DBH     DBH     DBH     DBH     DBH     DBH     CUT/    CUT/    TR  PRJ PRJ M1  M2  ST
TR.   TREE   PRED   FACTOR F.CTOR   CH     CH    TR  YEAR    OBS     OBS    PRED     OBS    PRED     OBS    DEAD    DEAD  STAT SPP DBH ST  ST  OBS TR
NUM   SPP.   1976   /ACRE  / CRE   INIT   CUR.  CC  GROWTH  INIT   MID 1   MID 1   MID 2   MID 2   FINAL  YR OBS  YR PRD  CUR GRP  CL O-P O-P FNL CL
                                                           1959    1964    1964    1969    1969
**************************************************************************************************************************************************
      1      2      16     13      5      3     4     6      7       8      11       9      12      10      14      15    1   2   3   4   5   6  7
**************************************************************************************************************************************************
  1   105  13.81   7.28   7.00   6.43   5.41   2   .27    9.80   11.00   11.05   11.90   12.17   12.90      0       0    1   1   1  11  11   1 20
  2   375   7.37   2.47   7.00   5.09   5.04   3   .19    6.00    7.20    6.99    8.40    7.37    9.70      0    1967    3   2   1  11  13   1 20
  3   105  17.89  12.22   7.00   6.40   5.36   2   .27   12.80   14.00   14.38   14.70   15.84   16.30      0       0    1   1   1  11  11   1 20
  4   105  13.35   6.87   7.00   7.42   6.38   2   .21    9.50   10.30   10.70   11.10   11.78   12.70      0       0    1   1   1  11  11   1 20
  5   105  16.37  10.23   7.03   4.80   3.77   3   .26   11.60   12.80   13.08   13.70   14.44       0   1972       0    1   1   1  11  11   3 20
  6   105  13.35   6.48   7.03   6.47   5.44   2   .21    9.50   10.70   10.70   11.70   11.78   12.10      0       0    1   1   1  11  11   1 20
  7   375   9.06   3.13   7.00   6.12   5.91   3   .17    5.90    6.90    6.88    7.80    7.81    9.00      0       0    1   2   1  11  11   1 20
  8   375   9.18   3.22   7.00   5.11   4.90   3   .17    6.00    7.00    6.99    8.00    7.93    9.30      0       0    1   2   1  11  11   1 20
  9    95   7.35   2.06   7.00   8.19   8.36   3   .17       0    5.30       0    6.00    6.16    6.70      0       0    1   3   1  40  11   1 20
 10   375   6.95   1.84   7.06   5.99   5.96   2   .14       0    5.20       0    6.30    5.93    7.10      0       0    1   2   1  40  11   1 20
 11    95   6.66   1.69   7.00   8.15   8.34   2   .18       0    5.60       0    6.50    6.48    7.20      0    1971    3   3   1  40  11   1 20
 12    95   7.81   2.33   7.00   7.99   7.83   2   .17       0    5.70       0    6.90    6.58    7.90      0       0    1   3   1  40  11   1 20
 13   105   6.24   1.49   7.00   3.79   3.43   3   .00       0       0       0    6.10       0    6.80      0       0    1   1   1   0  40   1 20
 14   375   6.92   1.63   7.00   5.89   5.87   3   .14       0       0       0    5.90       0    7.50      0       0    1   2   1   0  40   1 20
 15   375   6.08   1.41   7.00   4.96   4.95   4   .12       0       0       0    5.20       0    5.80      0       0    1   2   1   0  40   1 20

   NUMBER OF YEARS IN RU  = 17                                 BASAL AREA PER ACRE BY YEAR
      26.00    27.45    28.90    30.37    31.66    33.36    39.69    41.49    41.12    42.84    44.57    50.43    50.73    52.73    54.73    56.71
      58.69    60.65

   LISTING OF IP ARRAY
         1        2        3        4        5        6        7        8        9       10
         1        3        0       15        1        3        1        1        0        2
         5        1       26       59       64       69        1        1       61       41
        37       48        0        0        0        0        0        0        0        0
        59       64       69       76
      PLOT      1 CONSISTING OF    15 TREES COMPLETED     1     1

        1 PLOTS WITH       15 TREES FROM TEST OF UPDATE CHANGES USING CLOQUET FILE
```

TESTS

Rolfe A. Leary *and* **Jerold T. Hahn,**
Principal Mensurationists,
and **Roland G. Buchman,** *Principal Biometrician*

The purposes of this paper are (1) to report on tests of the design for assembling projection system parts, and (2) to report on tests of projection system reliability in various use domains. The use domains are many. For example, the forest type domain covers all Lake States types; we are interested in the projection system reliability for each type. Another domain is that of stand origin mode —plantation or natural. A third relates to geographical regions within the Lake States; does the system work better in one State than in others? For which types in which State? A fourth and particularly important domain is time. What is the time decay rate for projection system reliability? Other domains include stocking levels, sites, ages, species, sample plot types (fixed or variable radius) and mean stand diameters. Each domain or combination of domains warrants investigation. In this paper we report on item 1 above and the domains of (1) cover type, (2) geographical area, (3) time, and (4) sample plot type.

TESTING MODELS OF SYSTEM DYNAMICS

Mathematicians recognize two possible sources of error in predicting the future state of a dynamic system, such as a forest stand:

1. errors in estimating the initial state (initial conditions) of the system, and
2. errors in the mathematical model that characterizes system dynamics.

Errors in predicted future states may be caused by either of these sources or both may act in concert. Source (2) may be further broken down into:

(a) the mathematical equations are inappropriate for describing the process,

(b) the manner of combining the equations is incorrect, and

(c) the numerical constants obtained from calibration are not those governing the system.

An example of error type 1 for the projection system is when a person does not have an observed tree list but only a dot tally of trees on the plot by, say, 4-inch diameter classes.

An example of error type 2a is if the potential of trees to grow in diameter at breast height is not satisfactorily characterized by our potential function (Hahn and Leary 1979). Error type 2b is exemplified by a finding that the realized stand growth cannot be characterized by a product of a potential and modifier functions. Error type 2c is obvious; the numerical constants in use do not characterize the specific process under test.

Considerable effort has gone into developing theoretically sound mathematical equations for describing the process of forest growth. Hence error type 2a will not be evaluated in this paper. Such evaluations belong more properly in the separate papers decribing each equation. We will, however, evaluate the projection system to assess the magnitude of type 2b and 2c errors. We are not concerned with assessing type 1 errors—this is an inventory problem. We have prevented type 1 errors by using only permanent growth plot information. To assess type 2b errors we have used the permanent growth plot information available to calibrate the mathematical equations (Christensen *et al.* 1979). If the projection system performs well when checked against the calibration data base, it may be tested against an independent data base (i.e., one not used in calibration). By doing so, and by comparing predicted future states with observed future states (starting from a known past condition) we are able to assess the magnitude of type 2c errors. The validation data base for this comes from five sources:

(1) Cloquet Experimental Forest of the College of Forestry,[1] University of Minnesota;
(2) Chequamegon National Forest, Wisconsin;
(3) Nicolet National Forest, Wisconsin;
(4) A second-growth northern hardwood study (NH-2) on an experimental forest near Marquette, Michigan; and
(5) Hiawatha National Forest, Upper Michigan.

Figure 1 shows the geographical locations of sources. The fifth source, the Hiawatha National Forest, has an eastern and a western part. The distribution of plots by age class and site index class is shown in table 1.

METHODS AND PROCEDURES

To test both the method of assembling system parts and the applicability of the system to other Lake States forest conditions, the validation option was selected in the computer program that implements the projection system (Hahn and Brand 1979). Selecting this option causes the program to:

(1) read up to four sets of observed diameter measurements for each tree on a plot and store them for comparison with sets of predicted diameters,

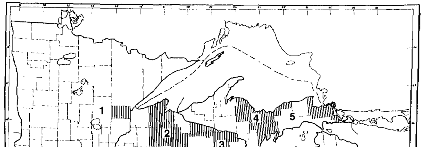


Figure 1.—*Geographical location of the five sources of permanent growth plot data used for independent tests of the projection system.*

---

[1] *We thank Dietmar Rose and the staff of the Cloquet Forestry Center for making past measurement data available and for assistance in making a measurement in 1976.*

Table 1.—*Distribution of 785 permanent forest growth plots used for model validation by age and site within 12 forest types in the Lake States (site class 30 ranges from 25 to 34, 40 from 35 to 44, etc.)*

JACK PINE

Age class	30	40	50	60	70	80	Total
	Site index class						
0-20		1		3			4
21-30		2			1		3
31-40			3	3	2		8
41-50		4	3	4	5		16
51-60	1	3	3	13	2		22
61-70			2	10	2		14
71-80			8	10			18
81-90			2	6			8
91-100			1				1
100+			1				1
Total							95

WHITE SPRUCE-BALSAM FIR

Age class	30	40	50	60	70	80	Total
0-20			2	2	2		6
21-30		1				1	2
31-40		2	2	1	1		6
41-50		2	4	7	2		15
51-60		3	5	2	2		12
61-70		1	6	6			13
71-80			5	5			10
81-90	1		3	2			6
91-100	1		1				2
100+	1	1					2
Total							74

TAMARACK

Age class	30	40	50	60	70	80	Total
0-20	1						1
41-50			1				1
51-60		2	2				4
61-70			1				1
71-80	1			1			2
81-90		1	2				3
91-100		4					4
100+		3					3
Total							19

OAK-HICKORY

Age class	30	40	50	60	70	80	Total
31-40					1		1
41-50			1	1	1		3
51-60			1	2			3
Total							7

RED PINE

Age class	30	40	50	60	70	80	Total
	Site index class						
21-30			2		1		3
31-40			1	5	9	1	16
41-50			1	5	2	1	9
51-60			3	3	2		8
61-70	1		5	9	1		16
71-80		2	6	4			12
81-90		1	3	2			6
91-100		1		1	1		3
100+		1	4	6			11
Total							84

BLACK SPRUCE

Age class	30	40	50	60	70	80	Total
0-20		1					1
31-40		3					3
41-50	3						3
51-60	1	1	1				3
61-70	1	4	1				6
71-80	5		1				6
81-90	2	1					3
91-100	1	4	1				6
100+	7		1				8
Total							39

LOWLAND HARDWOOD

Age class	30	40	50	60	70	80	Total
31-40					1		1
61-70			1				1
91-100		1					1
100+	1						1
Total							4

QUAKING ASPEN

Age class	30	40	50	60	70	80	Total
0-20	1	1	3	12	9	2	28
21-30			2	1	2		5
31-40			2	4	5		11
41-50		1	7	12	16	4	40
51-60		2	9	5	7		23
61-70			1	3	2		6
71-80				2			2
81-90			1				1
91-100			1				1
100+	1						1
Total							118

WHITE PINE

Age class	30	40	50	60	70	80	90	Total
	Site index class							
31-40		1		1	1			3
51-60			1	2	2			5
61-70			1	2				3
71-80		1	1					2
100+				1				1
Total								14

NORTHERN WHITE-CEDAR

Age class	30	40	50	60	70	80	90	Total
21-30			1					1
31-40	2	1		1				4
41-50	1		1					2
51-60	1	3	3					7
61-70	9	2	1					12
71-80	1	1						2
81-90	1							1
91-100	3							3
100+	10	3	1					14
Total								46

NORTHERN HARDWOOD

Age class	30	40	50	60	70	80	90	Total
0-20			1		2	1		4
21-30			1	1			1	3
31-40				1	7	2		10
41-50			3	11	22	1		37
51-60		1	9	25	11			46
61-70			4	13	105	1		123
71-80			1	3	1			5
91-100				1				1
100+	1		3	2	1			7
Total								236

PAPER BIRCH

Age class	30	40	50	60	70	80	90	Total
0-20			1	2				3
21-30	1				1			2
31-40			1		1	1		3
41-50		1	1	5	1			8
51-60	1	1	5	5	2			14
61-70			2	5	1			8
71-80			3	2	1			6
91-100			2					2
100+		1	1	1				3
Total								49

(2) use the first set of diameter measurements to start the projection,

(3) remove a tree from the projection tree list when it has been observed to have been cut,

(4) add a tree to the list of trees being projected when it is observed to be an ingrowth tree, and

(5) write an output file that contains observed and predicted diameter and status for each tree present on a plot at the first measurement. This is done for up to three points in time.

Ingrowth trees are not written on the output file (step 5) for comparing observations and predictions. They are included in the projection process, however, because in some cases ingrowth trees comprise a major part of the stand by the time of the final observation. Ignoring them could cause serious overestimation.

Analysis of the information produced at step 5 proceeds in two directions: (1) analysis for accuracy, and (2) analysis for precision.

The accuracy analysis is based on all the plots that occur in a forest type. We compare the predicted and observed mean stand diameter, trees per acre, and basal area per acre in a forest type by forming the ratio

$$\frac{\text{predicted mean stand attribute for the type}}{\text{observed mean stand attribute for the type}} \cdot$$

Clearly, ratios consistently greater than one indicate overprediction (positive bias), and those consistently less than one indicate underprediction (negative bias). Caution is needed in interpreting ratios where the observed attribute (the denominator of the fraction) is very small and both observed and predicted are expressed as integers.

The precision analysis is needed to assess the repeatability of the estimated ratios for each type. Here we compute for each plot in a forest type the ratio of predicted to observed variable (mean stand diameter, trees per acre, basal area per acre), the mean ratio over all plots, and a standard deviation of the ratios.

Conceivably the projection system could, at 10 years, have a positive bias of exactly 6 percent for every plot in a forest type. This would not be an accurate predictor but would be precise. Lack of accuracy can, of course, be corrected. Lack of precision cannot be easily corrected. Rather, it indicates a general inability of the underlying model to capture the essence of the processes underlying stand dynamics.

To test the method of assembling the parts of the system (error type 2b) we used selected forest properties from the calibration data base, and did an accuracy analysis only. To test the applicability of the numerical constants to other forests in the Lake States region (error type 2c) we used the five properties in the validation data base (fig. 1), and did both an accuracy and precision analysis. In both cases, we based our analysis on all plots in a property regardless of their stocking level; plots having as few as three or four trees were used as well as fully stocked plots. Clearly, failure to satisfactorily pass the test on method of combining system parts makes the other test largely academic. Finally, it should be noted that the plots are typed by the projection program (Hahn and Brand 1979), and that trees of several species may be present on a plot, not just trees of the species the type name bears.

TEST RESULTS AND DISCUSSION

This section is broken into two parts; the first gives the results of testing the manner of combining system parts, and the second tests the applicability of the complete system, including values of numerical constants, to other forests of the Lake States.

Combining the System Parts

The results of tests are shown in figure 2. The model seems to predict mean stand diameter better than either number of trees or basal area. This is not surprising, since an error in predicting number of trees—even if diameter is predicted exactly—will usually cause an error in predicting total stand basal area.

Even so, the three ratios for most species are gratifyingly close to one at most points in time. Several points can be made based on the contents of figure 2:

(1) Predictions for Minnesota jack pine are decidedly better than those for Lower Michigan when the evaluation is based on mean stand diameter and total stand basal area. However, the model estimates number of trees better for Lower Michigan stands.

(2) Predictions for Minnesota and Wisconsin red pine plantations and natural stands are quite good, although nearly all show ratios somewhat greater than one.

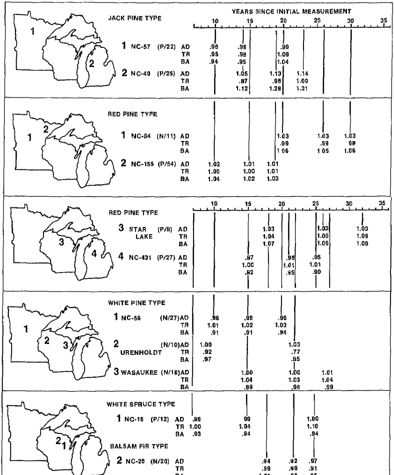
(3) White pine mean stand diameter is predicted fairly well. The mean d.b.h. for the Minnesota source is underestimated, while that for the Wisconsin source is somewhat overestimated. Urenholdt trees per acre after 22 years is off some (23 percent), but trees per acre at the other locations are quite good.

(4) Only one small plantation was available to calibrate the system for white spruce. Even after 25 years the predictions are good.

(5) Predicted values for the balsam fir type are less than the observed values. The average stand diameter prediction gets progressively worse for the first 18 years, but then improves over the remaining 7 years of observations.

(6) Both average diameter and number of trees are underestimated for the black spruce and northern white-cedar types. Combined, these greatly underestimate basal area. Interestingly, most of the prediction error occurs in the first 10- to 13-year period. Thereafter the prediction essentially parallels the observation.

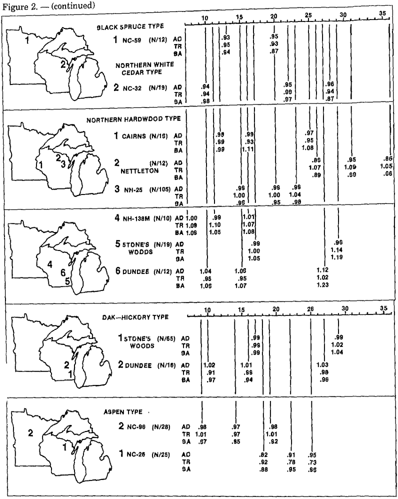
(7) Average diameter for test plots in the northern hardwood type is, for the most part, within ± 4 percent of the observed. An exception is the Dundee timber harvest forest, which overestimates average diameter by 12 percent after 27 years. Perhaps the severest test of the projection system occurred



gure 2.—*Test results of accuracy analysis of method of combining system parts. Each forest property is identified by name or number, plantation (P) or natural (N) stand and number of plots used in the comparison. Criteria for comparison are mean stand diameter for the type (AD), mean trees per acre for the type (TR), and mean stand basal area (BA) per acre for the type. Maps show approximate location of each property in Lake States. Ratios are computed as*

$$\frac{\textit{predicted mean stand attribute for a forest type}}{\textit{observed mean stand attribute for a forest type}}$$

Figure 2. — (continued)



with the Nettleton plots. The average stand diameter of the first measurement was between 1 and 2 inches. Even though projections were made for 36 years, the estimated mean stand diameter was within 4 percent of the observed.

(8) Predictions for the oak-hickory type, as found in southern Wisconsin, are reasonably good, although there is a tendency to overestimate mortality on the Dundee timber harvest forest.

(9) The Minnesota aspen type is reasonably well predicted for both average diameter and number of trees. The fact that basal area is underestimated so much is probably due to allocating too much of the diameter growth to small trees. The Wisconsin aspen comes from the same study as the balsam fir test. The 25 plots under test here probably had a large component of balsam fir. As with the 20 plots in the balsam fir type, mortality is overestimated and the average diameter is underestimated.

Tests on Independent Data Sets

Accuracy analysis

Five different data sets form the basis for these tests (fig. 1). The 266 Cloquet plots have been measured four times. The ratios of predicted to actual stand attributes is shown in table 2 for 5, 10, and 17 years after the initial measurement. Ratios

Table 2.—*Summary of Cloquet Experimental Forest test showing ratios of projected to actual mean plot diameter, trees per acre, and basal area per acre for the type totals converted to a per-acre basis (Cloquet plots are all $^{1}/_{7}$ acre, fixed-radius)*

Forest type	No. of plots	Years after initial measurement								
		5	10	17	5	10	17	5	10	17
		(ratio of predicted to actual mean plot diameter)			(ratio of predicted to actual number of trees per acre)			(ratio of predicted to actual basal area per acre)		
Jack pine	43	1.02	1.03	1.04	1.02	1.03	1.06	1.05	1.09	1.13
Red pine	53	.99	.99	.98	1.00	.99	.99	.99	.97	.96
White pine	6	1.00	.99	.96	1.04	1.11	1.20	1.02	1.03	1.01
W.spruce-fir	37	1.02	1.03	1.04	1.08	1.12	1.26	1.12	1.19	1.37
Black spruce	29	1.01	1.03	1.05	1.05	1.09	1.18	1.07	1.14	1.32
N.w.-cedar	15	1.01	1.02	1.04	.99	1.01	1.03	1.01	1.06	1.12
Tamarack	15	1.02	1.03	1.06	.97	.97	.97	1.01	1.04	1.09
Lowland hdwd.	2	1.00	1.01	1.01	1.13	1.07	1.39	1.09	1.00	1.26
N. hardwood	2	1.01	1.05	.92	1.00	1.00	1.33	1.02	1.10	1.20
Aspen	32	1.02	1.05	1.07	.93	.93	1.00	.97	.99	1.11
Paper birch	26	1.02	1.03	1.04	1.03	1.03	1.01	1.06	1.09	1.10

Table 3.—*Summary of NH-2 test projection for the northern hardwood type (NH-2 plots are $^{1}/_{10}$ acre, fixed-radius)*

Number of plots	Years after initial measurement								
	7	8	11	7	8	11	7	8	11
	(ratio of predicted to actual mean plot diameter)			(ratio of predicted to actual number of trees per acre)			(ratio of predicted to actual basal area per acre)		
37	1.00	—	.99	.99	—	.99	1.02	—	1.00
62	—	.99	—	—	.98	—	—	.97	—

Table 4.—*Summary of 10-year national forest test projections (all comparisons are based on repeated measurements of variable radius 10-point cluster plots)*

Forest type	Chequamegon				Nicolet				Hiawatha			
	No. of plots	D.b.h.	Trees	Basal area	No. of plots	D.b.h.	Trees	Basal area	No. of plots	D.b.h.	Trees	Basal area
Jack pine	7	1.04	1.02	1.09	5	1.06	1.18	1.23	20	1.09	1.03	1.16
Red pine	6	.99	1.11	1.00	6	1.02	.85	.96	12	.99	.94	.92
White pine	3	.96	1.09	.93	2	.93	.84	.87	3	.97	.99	1.00
W.spruce-fir	15	1.05	1.09	1.10	14	1.03	.91	1.05	8	1.02	1.53	1.16
Black spruce	2	1.13	.65	1.02	3	1.04	1.15	1.26	5	1.04	1.14	1.16
N.w.-cedar	6	1.00	1.00	1.02	12	1.02	.91	.96	14	1.04	.88	1.02
Tamarack	2	1.00	.75	.85	1	1.01	.95	1.01	—			
Oak-hickory	4	1.00	.70	.83	1	1.05	1.24	1.18	2	.95	.86	.87
Lowland hdwd.	2	1.00	.71	.99	—				—			
N. hardwood	45	1.01	.87	.95	54	1.00	.88	.94	36	1.01	.93	.96
Aspen	23	.98	.88	.98	44	1.02	.70	.97	17	1.04	.94	1.03
Paper birch	7	.96	1.18	1.09	6	.96	.72	.77	3	1.04	1.15	1.31
All types	122	1.01	.74	.95	148	1.01	.94	1.00	120	1.01	1.00	1.04

for the second growth northern hardwood study are shown in table 3. Thirty-seven plots had three measurements and 62 had only two measurements. The three national forests had two measurements only, so only one ratio is presented for them in table 4.

The Cloquet test shows 8 of 11 types have overestimates of mean d.b.h., 9 of 11 have overestimates of trees per acre and 10 of 11 have overestimates of basal area. Although the mean iameter overestimates are not serious, they are otable for their consistency and regular progresion; for example, 1 or 2 percent after 5 years, 2 or 3 ercent after 10 years, and 4, 5, or 6 percent after 17 years for white spruce-fir, black spruce, northern white-cedar, tamarack, and paper birch types. In fact, all types except red pine, white pine, and northern hardwoods have nearly the same progression of accuracy decay in predicting mean diameter. There is close agreement in both the type of error (over- or underestimate) and progression of accuracy decay when evaluating the system on number of trees. Exceptions appear to be the white pine, tamarack forest types and northern hardwoods. The ratios for predicted and observed basal area follow closely the estimates of mean diameter and number of trees.

The Cloquet results confirm earlier forecasts that the projection system would overestimate observed variables. This forecast was based on personal observations of the field remeasurement crew on the quantity of large shrubs and small trees present on Cloquet plots. Any competing vegetation that has affected growth of the trees on the plot but is not a part of the recorded observations will result in overestimates.

The major independent test for northern hardwoods was based on the repeated measurements of a study (NH-2) in Michigan's Upper Peninsula (table 3). The ratios are close to one in almost every case, and hence warrant little discussion.

The Chequamegon, Nicolet, and Hiawatha National Forest plot data cover a period of 10 years (table 4). These plots differ from the Cloquet and NH-2 plots in three ways: the plots are 10-point clusters of variable radius plots, the trees are measured down to 1.0 inch diameter, and the remeasurement occurred 10 years after the initial measurement. Most notable of the Chequamegon results are the relatively large overestimates of mortality in the black spruce, tamarack, oak-hickory, lowland hardwood, northern hardwood, and aspen types. One might discount overestimates in the first four types as chance occurrences because of the low plot numbers. We feel it is more likely because the mean plot diameters tend to be small, especially on the black spruce type, and because the number of trees represented by a small tree is larger than the number represented by a large tree (recall these are variable-radius plots). Thus, if the projection should assign a mortality status to a very small tree, it may mean that 100 trees per acre (its tree factor) are considered to have died. This points up one of the major differences in the way fixed- and variable-radius plots are projected.

The Nicolet also shows low estimates of numbers of trees for the aspen and paper birch forest types. The Hiawatha shows a 53-percent overestimate of trees per acre for the white spruce-fir type. Most of this difference is attributable to three of the eight plots. Because we are underestimating mortality on some plots by as much as 242 trees per acre in only 10 years, it is apparent that some variable other than competition is dominating the mortality process.

Precision analysis

The precision analysis, based on the Cloquet and NH-2 data sources, differs from the accuracy analysis in two respects:

(1) we consider only the ratio of predicted to actual mean stand diameter (ratios for number of trees and basal area have been computed but are not included here), and
(2) we consider the ratio for each individual plot in computing the mean and standard deviation of the ratios over time for for each measurement following the first.

Again, the analysis is organized by forest type. Figure 3 contains graphs for nine of the Cloquet types (lowland hardwood has been omitted), and figure 4 shows the mean and standard deviation at the two points in time for the northern hardwood type from NH-2.

Our concern with this analysis is with the pat-

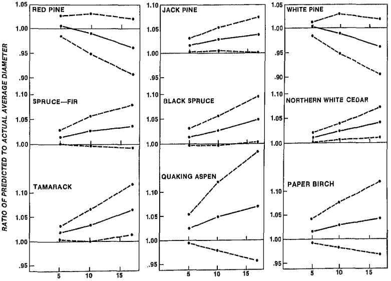
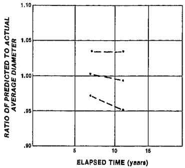


Figure 3.—*Pattern of ratios and standard deviation of ratios of predicted to actual mean plot diameter for nine forest types on the Cloquet Experimental Forest, Minnesota. Standard deviations are shown in dashed lines, means in solid lines.*

Figure 4.—*Pattern of mean ratios and standard deviation of ratios of projected to actual mean plot diameter for the northern hardwood type.*



tern created by plotting the mean ±1 standard deviation over time. Specifically, we look at (1) the closeness of the mean ratio to 1.00 at each time, and (2) the magnitude of the standard deviation and its relation to time since the initial measurement. Concern number 2 is particularly important.

Figures 3 and 4 show that all mean ratios are within ±7 percent of 1 after 17 and 11 years, respectively. For most types the ratios begin near 1 at 5 years and increase linearly to the value at 17 years. Jack pine, white spruce-fir, quaking aspen, and paper birch have noticeable concave patterns to the mean ratio. Tamarack and northern white-cedar show signs of a convex pattern. In all cases except the red and white pine types, extrapolation backward to the initial measurement would yield a ratio near 1.0.

Comparing the standard deviation magnitude shows that northern white-cedar is smallest, with black spruce, white spruce-fir, and jack pine also low. Quaking aspen has the largest standard deviation, followed by paper birch, red pine, and white pine.

The trend in standard deviation magnitude over me shows three patterns: (1) linear, (2) concave, nd (3) convex. We consider jack pine, paper birch, nd black spruce to be linear. Red pine, white pine, aspen, and white spruce-fir are judged concave. Tamarack and northern white-cedar appear to exhibit more of a convex pattern. With only two points, a trend cannot be assigned to the northern hardwood type (fig. 4).

The least desirable standard deviation trend type would be convex; linear would be better, and the most desirable would be concave. Trend type alone, however, is not sufficient to rate the goodness of projections by forest type, because it is confounded with standard deviation magnitude and the ratio trend itself. For example, we rate white pine, red pine, aspen, and white spruce-fir as having the best standard deviation trend type (concave), but none of these rates low on standard deviation magnitude. The northern white-cedar type shows a linear trend type and it has the lowest standard deviation. Because of these conflicting relations (fig. 3) it is not possible to say which type is projected the best (as measured by predicted vs. actual mean diameter). It is, however, possible to identify three candidates for "poorest" rating: quaking aspen, paper birch, and tamarack. A more definitive analysis could probably be made if numbers of trees were also included.

Table 5 summarizes the results of the precision analysis based on individual plot ratios of predicted to actual mean diameter for each forest type.

Table 5.—*Summary of precision analysis findings for projections of mean stand diameter (all results are based on Cloquet Forest plot data except the northern hardwood type, which is based on NH-2 from Upper Michigan)*

Forest type	Trend in ratio of projected to actual mean plot diameter	Comparative overall magnitude of standard deviation	Time trend in magnitude of standard deviation
Jack pine	linear	small	linear
Red pine	concave	medium	concave
White pine	concave	medium	concave
White spruce-fir	concave	medium	concave
Black spruce	linear	small	linear
N. white-cedar	convex	small	convex
Tamarack	convex	medium	convex
Northern hardwood	NA[1]	medium	NA[1]
Quaking aspen	concave	large	concave
Paper birch	linear	large	linear

[1] Not applicable because only two points in time are available.

SUMMARY

Tests of the projection system were based on comparisons of predicted and actual mean diameters, number of trees per acre, and basal area per acre for two data sets: (1) selected properties from the calibration data base, and (2) an independent data base. A summary of the important findings, organized around two central questions, is given below.

I. Is the method of combining system parts satisfactory?

1. The method of combining system parts (potential, modifier, allocation rule, mortality, etc.) is satisfactory.
2. Many of the calibration data base properties are projected very well for up to 30 years (fig. 2).
3. Properties with small trees are generally less well projected (e.g., NC-26, NC-32, NC-18, Nettleton, NC-59, fig. 2) than those with large trees.
4. Sizable growth rate differences for jack and red pine appear between Lower Michigan and Wisconsin-Minnesota. If validated by further checks, this indicates the need for separate coefficients for at least Lower Michigan.
5. Projections of average diameter are better than numbers of trees, which are better than basal areas.

II. Is the projection system applicable to other areas in the Lake States?

1. The projection system did a reasonably good job of projecting plots from Minnesota, Wisconsin, and Upper Michigan. These plots had not been used to calibrate the system.
2. Consistent positive bias (as much as 6 percent in 17 years) was found in predictions of mean d.b.h. for several cover types on the Cloquet Forest in Minnesota.
3. Variable-radius plots (10-point cluster design of Forest Survey) can be satisfactorily projected for 10 years, although mortality is sometimes overestimated for plots with many small trees.
4. No discernible deficiencies were found in projection capabilities for any species in the independent data sets as geographical location changed. It should be noted that we had no independent data set from Lower Michigan.
5. The standard deviation of the ratio of projected to actual mean plot diameter showed three types of trends when plotted over years since the original measurement: concave, linear, and convex. Northern white-cedar and tamarack forest types showed the least desired (convex) trend. Red pine, white pine, white spruce-fir, and quaking aspen showed the most desired (concave) trend.

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PRECIPITATION AND TEMPERATURE TRENDS IN THE LAKE STATES FOR SELECTED WEATHER STATIONS

Jerrilyn LaVarre Thompson, *Forestry Technician*

The purpose of this report is to summarize trends of precipitation and temperature at selected locations in the Lake States and discuss these trends and their possible impact on the calibration of a forest growth projection study. Substantial differences in the rainfall or temperature between the period used for calibration and future periods could have an effect on the accuracy of the projections.

Previous work on the impact of climate on tree growth has indicated a close correlation between precipitation and radial stem growth (Dils and Day 1952). The transition from wet years to dry years induces a gradual, rather than sharp, decrease in tree growth, reflecting a lag in soil moisture response to decreased rainfall. The reverse transition from dry to wet periods produces immediate growth response because soil moisture is replenished more rapidly than it is lost by evapotranspiration. Temperature also affects tree growth, primarily because it regulates growth initiation in the spring (Dils and Day 1952). However, we have made no attempt to correlate climate data and tree growth in this report.

DATA SOURCES

Most of the data in this report were obtained from records provided by the Minnesota State Climatology Office, 1903 Hendon Avenue, St. Paul, Minnesota 55108. Additional data were obtained through correspondence with Forest Service personnel in Michigan.

Precipitation and temperature data for the 6-month period, April through September, were compiled by month for 11 weather stations throughout Minnesota, Michigan, and Wisconsin from the year of each station's establishment through 1976 (fig. 1). The criterion for weather station selection was the station's proximity to permanent studies of forest growth.

In all but two cases, both precipitation and temperature were recorded at each station. At the Cass Lake, Minnesota station, only precipitation was recorded; at the Virginia, Minnesota station, temperature was the only variable recorded.

Precipitation was recorded by month at each station from April through September from the year of station establishment through 1976. Temperature was not recorded in the annals monthly until 1931; consequently, the values prior to 1931 are the April through September averages from the date of station establishment through 1930.

Minnesota and Wisconsin temperature values were recorded by month from 1931 through 1976. Michigan monthly temperature values were not recorded until 1941 at Cadillac, and 1951 at Allegan, Grayling, and East Tawas. No temperature information was available for Grayling or East Tawas until 1931; from 1931-1951, only the April-through-September average was available. April-through-September temperature averages were recorded for Cadillac and Allegan from station establishment to 1941 and 1951, respectively.

GROWING SEASON

Prior to discussing the data analysis, the term "growing season" as used in this text should be defined. Usually the growing season is described as the period between the last killing frost in the spring and the first killing frost in the fall. Some factors that determine whether a plant will be killed by frost are species, age, time of year, and duration and rate of freezing (Moran and Morgan

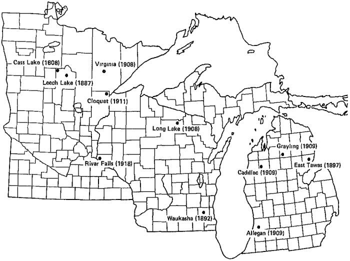

Figure 1.—*Location and date of establishment of meteorological stations. Monthly (April to September) precipitation and temperature information was recorded from each station, except Cass Lake, where only precipitation was recorded, and Virginia, where only temperature was recorded.*

1976). The Weather Bureau commonly designates the growing season as the "freeze-free" period, when minimum temperatures are greater than 32°F. (0°C.). Another measure employs a 28°F. (-2°C.) free period, which accommodates for local temperature variations.

In the Lake States the frost-free period generally extends from April through September. Moran and Morgan (1976) cite the three Wisconsin stations used in this study as having 28°F. free periods that range from 110 to 195 days. The Minnesota stations, Virginia, Cloquet, and Leech Lake, have 28°F. free seasons that extend from 104 to 159 days (Baker and Strub 1963). While most tree growth occurs in June and July, it does begin as early as April and can continue until September. Therefore, in this study we have defined the growing season as the 180-day period from April through September, to include all tree growth occurring in any year.

DATA ANALYSIS METHODS AND RESULTS

To prepare the information for analysis, several steps were taken. The monthly precipitation data were added to obtain growing season rainfall totals. Average temperatures for each growing season were also calculated.

Five-year averages for both precipitation and temperature were calculated for each station from its date of establishment through 1976 (figs. 2 and 3). "Even" 5-year intervals were used for ease of manipulation (i.e., 1901-1905, 1906-1910, etc.). The first average for a station may include more than five yearly measurements to accommodate this scheme. For example, Allegan, Michigan precipitation data began in 1909, but the first "even" interval is 1911-1915; consequently, the first interval is 7 years, from 1909-1915. All subsequent averages are 5 years, except for the last interval,

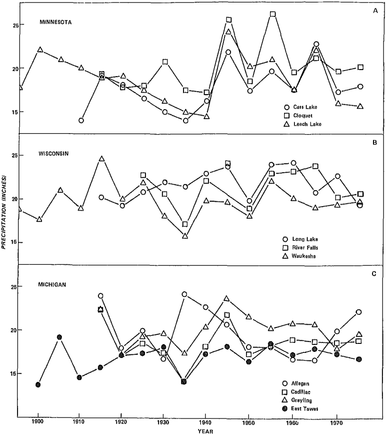

Figure 2.—*April-to-September precipitation trends in the Lake States. Five year averages from selected weather stations were plotted at the endpoint of each interval.*

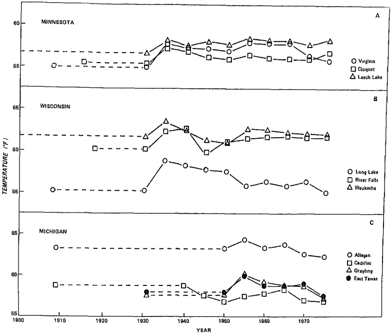

Figure 3.—*April-to-September temperature trends in the Lake States. Five year averages from selected weather stations were plotted at the endpoint of each interval. Dashed lines indicate the average for periods when monthly or yearly averages were missing.*

1971-1976. The average for an interval is plotted at the endpoint of that interval.

Trends in precipitation and temperature were analyzed for three periods: (1) station establishment through 1976; (2) station establishment through 1935; and (3) 1936-1976. The period 1936-1976 encompasses the interval when field measurements for the forest growth projection study were taken, and is referred to as the calibration period (Christensen *et al.* 1979). In addition, the months of June and July were analyzed separately for the calibration period. Soil moisture is critical during these months, because it is then that soil moisture deficits are first apparent, and it is also the time when most growth occurs (Fraser 1962).

Yearly fluctuations in precipitation for the various weather stations are large, yet there are some notable trends. For example, in Minnesota from 1911 through 1976, growing season precipitatio at the Leech Lake, Cass Lake, and Cloquet stations increased slightly overall, as shown by linea regression analysis. Until 1940 a decrease in rainfall was recorded for all three stations. Cass Lake showed a marked increase in precipitation from 1941 to 1976 (fig. 2a). Leech Lake showed an increase in rainfall from 1940 to 1945, but a gradual decline from 1946 to 1976.

Cloquet showed a decrease in precipitation from 1940 to 1976. From 1936 to 1976 for June-July rainfall gradually increased at all three Minnesota stations.

In Wisconsin, precipitation increased from 1918 to 1976. From station establishment to 1976, the Long Lake and River Falls stations showed increasing rainfall amounts, but the Waukesha station recorded a decrease (fig. 2b). The period from station establishment to 1935 was generally drier than the period from 1936-1976. The 1936-40 interval marked an increase in rainfall for a decade. Since 1940, all Wisconsin weather stations have recorded an overall decrease in precipitation. For the June-July analysis, Long Lake and River Falls showed a decrease in the amount of rainfall, while Waukeaha showed an increase.

In Michigan the overall trend from 1909 to 1976 was one of decreasing rainfall (fig. 2c). From station establishment through 1976, Allegan and Cadillac exhibited this downward trend; however, Grayling and East Tawas showed an increase in rainfall for the same period. From station establishment to 1935, East Tawas experienced progressively more rainfall, while the other Michigan stations experienced a drying trend. During the calibration period these trends continued. However, the June-July analysis showed decreasing precipitation at Grayling and increased precipitation at the other stations.

In the Lake States overall, the period from the various stations' establishment to 1935 was generally drier than the calibration period (1936-1976). Following 1935, which marked the end of the "Dust Bowl" era, precipitation increased for a decade throughout the Lake States. This was followed by a general drying trend through the 1976 growing season at all stations in the study, except at East Tawas and Cass Lake.

The average precipitation for the Lake States from station establishment through 1976 was 19.5 inches. From station establishment through 1935 the average was 18.7, and for the calibration period, 1936-1976, the average was 20.0 inches. Wisconsin had the highest averages: 21.0, 20.0, and 21.4 inches for the periods from station establishment-1976, station establishment-1935, and 1936-1976, respectively. Minnesota had an overall average of 18.7 inches, a station establishment-1935 average of 17.4, and a calibration period average of 19.5 inches. Michigan averaged 18.9 inches overall, 18.6 inches from station establishment-1935, and 19.1 inches during the calibration period.

Temperature trends for the study period did not fluctuate as much as precipitation patterns. Station temperature values were fairly constant through time, but varied by location. Allegan had the highest calibration period average, 63.2°F. (17.3°C.), followed by Waukesha and River Falls with 62.0°F. (16.6°C.) and 61.5°F., (16.4°C.), respectively. Cloquet had the lowest average, 56.3°F. (13.4°C.).

Because 5-year averages in Minnesota and Wisconsin were not available prior to 1931, and between 1941 and 1951 in Michigan, the information on temperature trends is limited to the calibration period.

Each State showed a slight cooling trend for the calibration period when linear regression analysis techniques were applied. On a station-by-station basis, however, only six stations exhibited a cooling trend. The other four stations— Leech Lake, River Falls, Waukesha, and Cadillac— indicated slight warming trends (fig. 3).

When June-July temperatures were examined, they were similar to the growing season trends at all stations except Cloquet, which showed gradually increasing June-July temperatures.

The average Lake States growing-season temperature for the period from station establishment to 1976 and for the calibration period was 58.9°F. (14.9°C.). The corresponding averages for these two time periods by State are as follows: Wisconsin— 60.1°F. (15.6°C.), and 60.0°F. (15.5°C.); Michigan—59.5°F. (15.3°C.) for both periods; and Minnesota—57.0°F. (13.9°C.) and 57.1°F. (13.9°C.).

Although the data indicate downward trends for both precipitation and temperature during the growing season and an increase in precipitation for the months of June and July (table 1), these trends may not continue into the future.

IMPLICATIONS OF FINDINGS

Precipitation and temperature are external factors affecting tree growth. They fluctuate both yearly and over longer time periods.

It is logical to assume that short-term fluctuations of both precipitation and temperature are normal and that their overall effect on forest

Table 1.—*Comparative growing season precipitation and temperature trends from Station establishment to 1976 and 1936-1976 (for the period 1936-1976, both the April to September and June and July trends are summarized)*

	Minnesota			Wisconsin			Michigan			
	Cass Lake	Cloquet	Leech Lake	Long Lake	River Falls	Waukesha	Allegan	Cadillac	Grayling	East Tawas
PRECIPITATION TRENDS										
Station establ.-1976 April-Sept.	increase	increase	decrease	increase	increase	decrease	decrease	decrease	increase	increase
1936-1976 April-Sept.	increase	decrease	decrease	decrease	decrease	decrease	decrease	decrease	decrease	increase
June-July	increase	increase	increase	decrease	decrease	increase	increase	increase	decrease	increase
TEMPERATURE TRENDS[1]										
Station establ.-1976 April-Sept.	increase[2]	increase	increase	decrease	increase	increase	decrease	decrease	increase	increase
1936-1976 April-Sept.	decrease[2]	decrease	increase	decrease	increase	increase	decrease	increase	decrease	decrease
June-July	decrease[2]	increase	increase	decrease	increase	increase	decrease	increase	decrease	decrease

[1] Michigan temperature trends from 1941-1976 at Cadillac, and from 1951-1976 at other stations

[2] Virginia recording station.

growth is minimal. These fluctuations are reflected in seasonal growth variation. During years of abundant rainfall and warm temperatures, individual trees or stands of trees put on more growth. Conversely, sparse rainfall and cold or extremely hot temperatures inhibit growth (Diller 1935). If these periods of highs or lows are prolonged, a marked difference in growth rates would be expected to follow.

Since the projection system is calibrated on measurements taken between 1938 and 1976, a very favorable period for tree growth in the Lake States, its use for projections into periods of unknown precipitation and temperature must be made with caution. If rainfall and temperature are higher in the future than during the calibration period, the probable result would be underestimation of tree growth by the models. If, on the other hand, the period following calibration is dry and cool or dry and exceptionally warm for an extended period of time, the models would probably overestimate actual growth.

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☆ U.S. GOVERNMENT PRINTING OFFICE: 1979-666-932/307 REG.#6

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